

Section 5.4 Solving Trigonometric Equations

i) $\cos(2x + 50^\circ) = \frac{\sqrt{3}}{2}$ $-180^\circ \leq x \leq 180^\circ$

Let $w = 2x + 50^\circ$

$$\cos w = \frac{\sqrt{3}}{2}$$



$$w = 30^\circ + 360^\circ k$$

$$2x + 50^\circ = 30^\circ + 360^\circ k$$

$$2x = -20^\circ + 360^\circ k$$

$$x = -10^\circ + 180^\circ k$$

$$\begin{array}{l} k=0 \quad x = -10^\circ \\ k=1 \quad x = 170^\circ \end{array}$$

$$w = 330^\circ + 360^\circ k$$

$$2x + 50^\circ = 330^\circ + 360^\circ k$$

$$2x = 280^\circ + 360^\circ k$$

$$x = 140^\circ + 180^\circ k$$

$$\begin{array}{l} k=1 \quad x = -40^\circ \\ k=0 \quad x = 140^\circ \end{array}$$

ii) $4\cos^2 x + 4\cos x + 1 = 0$

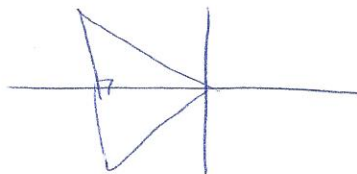
$$(2\cos x + 1)(2\cos x + 1) = 0$$

$$\cos x = -\frac{1}{2}$$

$$x = 120^\circ + 360^\circ k$$

$$k=0, x = 120^\circ$$

$$0^\circ \leq x \leq 360^\circ$$



$$x = 240^\circ + 360^\circ k$$

$$k=0, x = 240^\circ$$

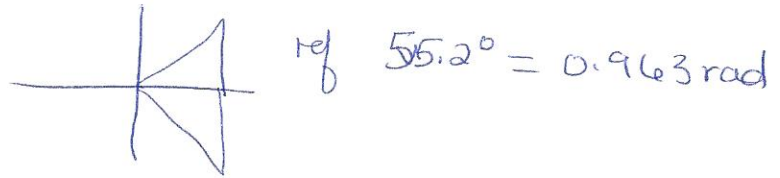
$$\text{ii) } 7 \cos(3x - 18) = 4$$

$$0 \leq x \leq 2\pi$$

$$\text{let } w = 3x - 18$$

$$7 \cos w = 4$$

$$\cos w = \frac{4}{7}$$



$$w = 0.963 + 2\pi k$$

$$3x - 18 = 0.963 + 2\pi k$$

$$3x = 18.963 + 2\pi k$$

$$x = 6.3 + \frac{2\pi k}{3}$$

$k \approx 2.09$

$$k = -1 \quad x = 4.21$$

$$k = -2 \quad x = 2.12$$

$$k = -3 \quad x = 0.03$$

$$w = 5.32 + 2\pi k$$

$$3x - 18 = 5.32 + 2\pi k$$

$$3x = 23.32 + 2\pi k$$

$$x = 7.8 + \frac{2\pi k}{3}$$

$$k = -1 \quad x = 5.71$$

$$k = -2 \quad x = 3.62$$

$$k = -3 \quad x = 1.53$$

$$\text{iv) } \sin\left(\frac{\pi}{4}(x-6)\right) = 0.5$$

$$\text{Let } w = \frac{\pi}{4}(x-6)$$

$$\sin w = 0.5$$

$$w = \frac{\pi}{6} + 2\pi k$$

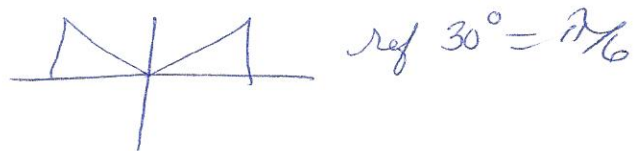
$$\frac{\pi}{4}(x-6) = \frac{\pi}{6} + 2\pi k$$

$$x-6 = \frac{4}{6} + 8k$$

$$x = \frac{20}{3} + 8k$$

$$k = -1 \quad x = -1.3 \text{ No!}$$

$$0 \leq x \leq 2\pi$$



$$w = \frac{5\pi}{6} + 2\pi k$$

$$\frac{\pi}{4}(x-6) = \frac{5\pi}{6} + 2\pi k$$

$$x-6 = \frac{20}{6} + 8k$$

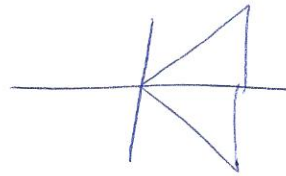
$$x = \frac{28}{3} + 8k$$

$$k = -1 \quad x = 1.3$$

$$v) \quad 8 \cos(2x-5) = 3$$

$$\text{let } w = 2x - 5$$

$$\cos w = 3/8$$



$$\text{ref } 67.97^\circ \\ = 1.19 \text{ rad}$$

$$w = 1.19 + 2\pi k$$

$$w = 5.09 + 2\pi k$$

$$2x - 5 = 1.19 + 2\pi k$$

$$2x - 5 = 1.19 + 2\pi k$$

$$2x = 6.19 + 2\pi k$$

$$2x = 10.09 + 2\pi k$$

$$x = 3.10 + \pi k$$

$$x = 5.05 + \pi k$$

vii)

$$5.2 \sin 45(x+8^\circ) - 1 = -3$$

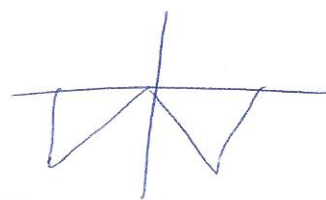
$$\text{let } w = 45(x+8^\circ)$$

$$5.2 \sin w - 1 = -3$$

$$5.2 \sin w = -2$$

$$\sin w = -2/5.2$$

$$\sin w = -0.3846$$



$$\text{ref } 22.6^\circ$$

$$w = 202.6^\circ + 360^\circ k$$

$$w = 337.4^\circ + 360^\circ k$$

$$45(x+8^\circ) = 202.6^\circ + 360^\circ k$$

$$45(x+8^\circ) = 337.4^\circ + 360^\circ k$$

$$x+8^\circ = 4.5^\circ + 8^\circ k$$

$$x+8^\circ = 7.5^\circ + 8^\circ k$$

$$x = -3.5^\circ + 8^\circ k$$

$$x = -0.5^\circ + 8^\circ k$$

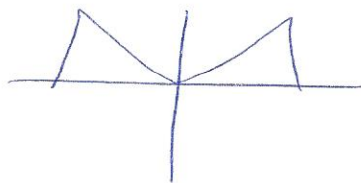
$$v(ii) \quad 0 = -3\sin 2x + 2$$

$$3\sin 2x = 2$$

$$\sin 2x = \frac{2}{3}$$

$$\text{let } w = 2x$$

$$\sin w = \frac{2}{3}$$



$$\begin{aligned} \text{ref } 41.8^\circ \\ &= 0.729 \text{ rad} \\ &= 0.73 \text{ rad} \end{aligned}$$

$$w = 0.73 + 2\pi k$$

$$2x = 0.73 + 2\pi k$$

$$x = 0.365 + \pi k$$

$$w = 2.4 + 2\pi k$$

$$2x = 2.4 + 2\pi k$$

$$x = 1.2 + \pi k$$

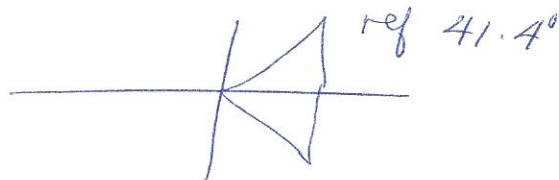
$$v(iii) \quad 4\cos(x - 45^\circ) + 7 = 10$$

$$\text{let } w = x - 45^\circ$$

$$4\cos w + 7 = 10$$

$$4\cos w = 3$$

$$\cos w = \frac{3}{4}$$



$$w = 41.4^\circ + 360^\circ k$$

$$x - 45^\circ = 41.4^\circ + 360^\circ k$$

$$x = 86.4^\circ + 360^\circ k$$

$$w = 318.6^\circ + 360^\circ k$$

$$x - 45^\circ = 318.6^\circ + 360^\circ k$$

$$x = 363.6^\circ + 360^\circ k$$