1. Write the mapping notation to describe how the graphs of the following functions can be obtained from the graph of $y = f(x)$.

a) $y - 3 = f(5x)$

$y = f(5x) + 3$ $\qquad$ $(x, y) \rightarrow (\frac{1}{5}x, y+3)$

b) $2y - 6 = f(4(x+1)$

$2(y-3) = f(4(x+1))$

$y - 3 = \frac{1}{2} f(4(x+1))$ $\qquad$ $(x, y) \rightarrow (\frac{1}{4}x - 1, \frac{1}{2}y + 3)$

$y = \frac{1}{2} f(4(x+1)) + 3$

c) $y = f(3x+6) + 1$

$y = f(3(x+2)) + 1$ $\qquad$ $(x, y) \rightarrow (\frac{1}{3}x - 2, y+1)$

2. Write the equation of the transformation in the form $y = af(b(x-h)) + k$ after the transformations described.

a) $y = g(x)$ is translated 4 units down, 3 units to the left and horizontally stretched by a factor of 5.

VT = -4

HT = -3

HS = 5

$y = f(\frac{1}{5}(x+3)) - 4$

b) $y = g(x)$ is translated 2 units up, 5 units to the right, reflected in the x-axis and vertically stretched by a factor of 3.

VT = 2

HT = 5

ref x axis

VS = 3

$y = {}^{-}f(x-5) + 2$

3. The mapping rule $(x,y)\rightarrow(2x-1,y+3)$ is applied to the function $y=f(x)$. What is the equation of the resulting function?

A) $y=f(2(x-1))+3$
B) $y=f(2(x+1))-3$
C) $y=f\left(\frac{1}{2}(x-1)\right)-3$
D) $y=f\left(\frac{1}{2}(x+1)\right)+3$

$HS=2$
$b=\frac{1}{2}$

HT 1 left
VT 3 up

4. The domain of $y=f(x)$ is $\{x \mid -4\leq x\leq 8, x\in R\}$ and the range is $\{y \mid -6\leq x\leq 12, x\in R\}$. What are the domain and range of $g(x)=\frac{1}{3}g(2x)$?

$g(x)=\frac{1}{3}g(2x)$
$VS=\frac{1}{3}$
$HS=\frac{1}{2}$

Domain: $-2\leq x\leq 4$
Range: $-2\leq y\leq 4$

5. The point (-4, 10) lies on the graph of $f(x)$. What is the image point on the graph of $y=-3f(2x-6)+1$?

$y=-3f(2(x-3))+1$
$(x,y)\rightarrow(\frac{1}{2}x+3,\ -3y+1)$
$x=-4$ $y=10$

point $(1,-29)$

6. Consider the function $f(x)=(x+4)(x-5)$. What are the zeros of the function if the graph is transformed by a horizontal stretch factor of 3 and reflected over the *y*-axis?

$HS=3$
reflected over y

A) $(-12,0)$ *and* $(15,0)$
B) $\left(-\frac{4}{3},0\right)$ *and* $\left(\frac{5}{3},0\right)$
C) $(12,0)$ *and* $(-15,0)$
D) $\left(\frac{4}{3},0\right)$ *and* $\left(-\frac{5}{3},0\right)$

$(x,y)\rightarrow(-3x,\ y)$

$(-4,0)\rightarrow(12,0)$
$(5,0)\rightarrow(-15,0)$

7. If the function $y = f(x)$ is horizontally stretched by a factor of $\frac{1}{4}$ and translated 5 units to the left and 1 unit downward, what is the new transformed equation?

A) $y = f\left(\frac{1}{4}(x-5)\right)-1$

B) $y = f\left(\frac{1}{4}(x+5)\right)+1$

C) $y = f(4(x-5))-1$

D) $y = f(4(x+5))-1$

$HS = 1/4 \Rightarrow b = 4$

$HT = -5$

$VT = -1$

8. What is the horizontal stretch factor of $\frac{1}{2}y = f(-5x)$?

A) -5

B) $-\frac{1}{5}$

C) $\frac{1}{5}$

D) 5

$y = 2f(-5x)$

reflection over y axis

$HS = 1/5$

9. What is the horizontal translation of the transformed function $y = 2f(-3x+6)+1$?

A) 6 units left

B) 2 units left

C) 2 units right

D) 6 units right

$y = 2f(-3(x-2))+1$

$HT = 2$ right

10. What is the vertical translation of the transformed function $3y - 6 = f(x+6)$?

A) 6 units up

B) 6 units down

C) 2 units up

D) 2 units down

$3(y-2) = f(x+6)$

$y-2 = \frac{1}{3}f(x+6)$

$y = \frac{1}{3}f(x+6)+2$

$VT = 2$ up

11. The point (a,b) is on the graph of $y = f(x)$. What are the coordinates of the image of this point on the graph of $y + b = f(x+1)$?

A) $(a-1, 2b)$

B) $(a+1, 2b)$

C) $(a-1, 0)$

D) $(a+1, 0)$

$y = f(x+1) - b$

$(x, y) \rightarrow (x-1, y-b)$

$x = a$ $\quad y = b$

12. Which mapping rule would map the function $y = f(x)$ onto the function $y = f\left(-\frac{1}{3}x + 3\right)$?

A) $(x, y) \rightarrow (-3x+1, y)$

B) $(x, y) \rightarrow (-3x+9, y)$

C) $(x, y) \rightarrow \left(-\frac{1}{3}x+1, y\right)$

D) $(x, y) \rightarrow \left(-\frac{1}{3}x+9, y\right)$

$y = f\left(-\frac{1}{3}x + 3\right)$

$y = f\left(-\frac{1}{3}(x-9)\right)$

HS = 3

reflection over y axis

HT = 9 right

13. The transformation $y = -3f(4(x-1)) + 2$ is best described as:

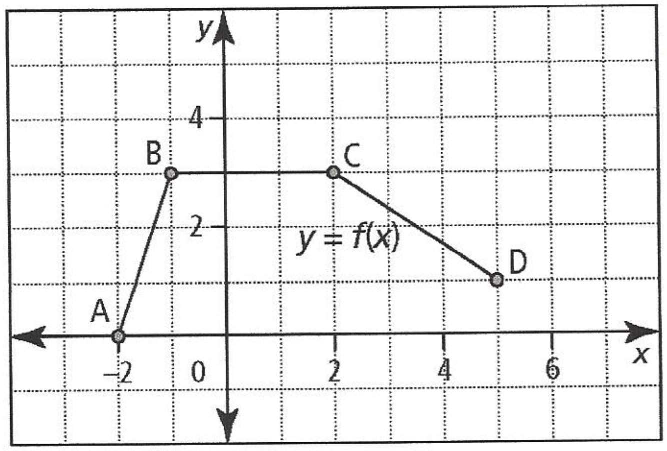
A) Reflection about the x- axis; a vertical stretch factor of 3 and a horizontal stretch factor of 4; translation 1 unit to the left and 2 units up.

B) Reflection about the x- axis; a vertical stretch factor of 3 and a horizontal stretch factor of $\frac{1}{4}$; translation 1 unit to the right and 2 units up.

C) Reflections about the y- axis; a vertical stretch factor of 3 and a horizontal stretch factor of $\frac{1}{4}$; translation 1 unit to the right and 2 units up.

D) Reflections about the y- axis; a vertical stretch factor of 3 and a horizontal stretch factor of 4; translation 1 unit to the right and 2 units up.

14. Consider the graph of $y = f(x)$.



Use the function $y - 5 = f(-\frac{1}{2}(x+3))$ to state the coordinates of the image points A′, B′, C′, and D′.

$y = f(-\frac{1}{2}(x+3)) + 5$

$(x, y) \rightarrow (-2x - 3, y + 5)$

$A(-2, 0) \rightarrow A'(1, 5)$

$B(-1, 3) \rightarrow B'(-1, 8)$

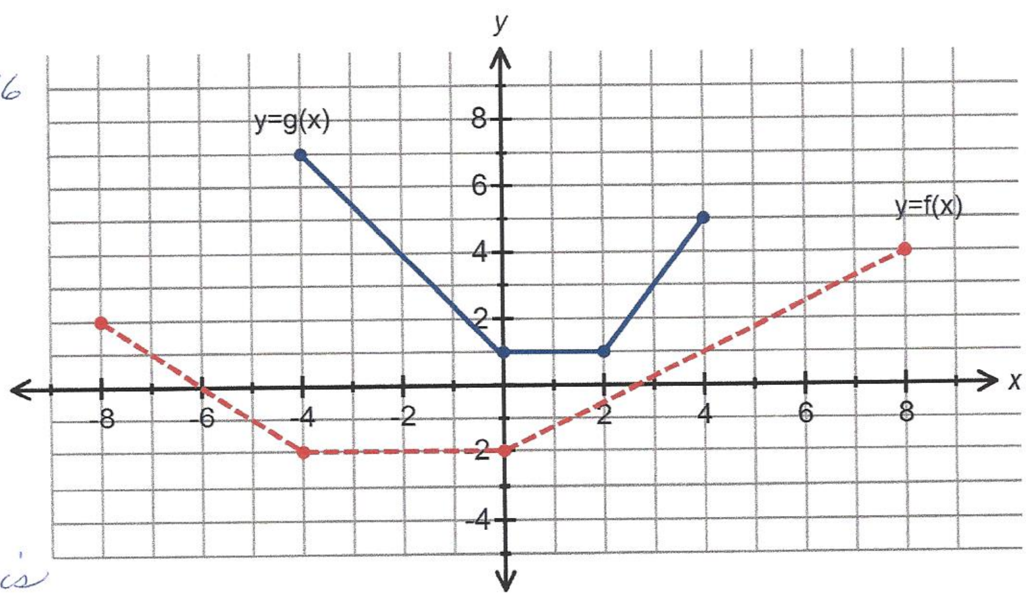
$C(2, 3) \rightarrow C'(-7, 8)$

$D(5, 1) \rightarrow D'(-13, 6)$

15. Determine the equation of the transformed graph $y = af(b(x-h)) + k$ given the graph of $y = f(x)$. original

domain original $[-8, 8] = 16$
domain new $[-4, 4] = 8$

$HS = \frac{8}{16} = \frac{1}{2}$



range original $[-2, 4] = 6$
range new $[1, 7] = 6$

$VS = \frac{6}{6} = 1$

reflection over y axis
$y = f(-x)$

original point $(8,4) \underset{HS}{\rightarrow} (4,4) \underset{VS}{\rightarrow} (4,4) \underset{ref}{\rightarrow} (-4,4)$

3 units up
$(-4, 4) \rightarrow (-4, 7)$

$VT = 3$
$HT = 0$

new point $(-4, 7)$

$\boxed{y = f(-2x) + 3}$

16. What is the inverse of $y = 2x^2 - 8$.

A) $y = \frac{x^2 + 8}{2}$

B) $y = \pm\sqrt{\frac{x+8}{2}}$

C) $y = \pm\sqrt{x+4}$

D) $y = 8 \pm \sqrt{\frac{x}{2}}$

$x = 2y^2 - 8$

$x + 8 = 2y^2$

$\frac{x+8}{2} = y^2$

$\pm\sqrt{\frac{x+8}{2}} = y$

17. Algebraically determine the equation of the inverse of $f(x) = 2x^2 + 8x + 1$. Identify a restricted domain for which the function has an inverse that is also a function.

$y = 2x^2 + 8x + 1$

$x = \frac{-b}{2a} = -2$

$y = -7$

$a = 2$

$y = 2(x+2)^2 - 7$

Inverse

$x = 2(y+2)^2 - 7$

$\frac{x+7}{2} = (y+2)^2$

$\pm\sqrt{\frac{x+7}{2}} = y + 2$

$y = -2 \pm \sqrt{\frac{x+7}{2}}$

Restriction

$y = 2(x+2)^2 - 7 \quad x \geq -2$

$f^{-1}(x) = -2 + \sqrt{\frac{x+7}{2}}$

or

$y = 2(x+2)^2 - 7 \quad x \leq -2$

$f^{-1}(x) = -2 - \sqrt{\frac{x+7}{2}}$

18. Given the graph of the function $y = f(x)$ below, sketch the inverse graph of $y = 3f(-2(x-1)) + 1$

$(x, y) \Rightarrow (-\frac{1}{2}x + 1, 3y + 1)$

$(-2, 4) \Rightarrow (2, 13) \Rightarrow (13, 2)$

$(0, 4) \Rightarrow (1, 13) \Rightarrow (13, 1)$

$(2, 2) \Rightarrow (0, 7) \Rightarrow (7, 0)$

$(4, 4) \Rightarrow (-1, 13) \Rightarrow (13, -1)$

$(6, 6) \Rightarrow (-2, 19) \Rightarrow (19, -2)$

original $\Rightarrow$ Transformed $\Rightarrow$ Inverse