Math 3200 Trigonometric Identities

#1a) $\sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ$
$= \sin(75^\circ + 15^\circ)$
$= \sin(90^\circ)$
$= 1$

b) $\cos\frac{5\pi}{12}\cos\frac{\pi}{12} - \sin\frac{5\pi}{12}\sin\frac{\pi}{12}$
$= \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right)$
$= \cos\left(\pi/2\right)$
$= 0$

1c) $\sin\left(\frac{11\pi}{12}\right) = \sin(165^\circ) = \sin(120^\circ + 45^\circ)$
$= \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ$
$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$
$= \frac{\sqrt{6} - \sqrt{2}}{4}$

1d) $\frac{\tan\frac{2\pi}{3} + \tan\frac{\pi}{12}}{1 - \tan\frac{2\pi}{3}\tan\frac{\pi}{12}} = \tan\left(\frac{2\pi}{3} + \frac{\pi}{12}\right)$
$= \tan(120^\circ + 15^\circ)$
$= \tan(135^\circ)$
$= -1$

#2 $2\sin\frac{\pi}{3}\cos\frac{\pi}{3} \Rightarrow 2\sin A\cos A = \sin 2A$
$= \sin(2 \cdot \pi/3)$
$= \sin(2\pi/3)$
$= \sqrt{3}/2$

#3. $\csc x \cos x = \cot x$
$\frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x}$

$\sin x \neq 0$
$x \neq 0^\circ, 180^\circ, 360^\circ$
$x \neq 0^\circ + 180^\circ k,\ k \in I$

#4. $\cot(-165°) = \dfrac{1}{\tan(-165°)}$

$= \dfrac{1}{\frac{\sqrt{3}-1}{1+\sqrt{3}}}$

$= \dfrac{1+\sqrt{3}}{\sqrt{3}-1} \cdot \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$= \dfrac{\sqrt{3}+1+3+\sqrt{3}}{3-1}$

$= \dfrac{2\sqrt{3}+4}{2}$

$= \sqrt{3}+2$

$\tan(-165°) = \tan(-120° + -45°)$

$= \dfrac{\tan(-120°) + \tan(-45°)}{1 - \tan(-120°)(\tan -45°)}$

$= \dfrac{\sqrt{3} + -1}{1 - (+\sqrt{3})(-1)}$

$= \dfrac{\sqrt{3}-1}{1+\sqrt{3}}$

#5. $\cos 75° = \cos(45° + 30°)$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

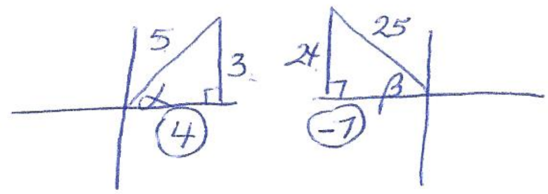
$= \left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{2}\right)$

$= \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right)$

$= \dfrac{\sqrt{6}}{4} - \dfrac{\sqrt{2}}{4}$

$= \dfrac{\sqrt{6}-\sqrt{2}}{4}$

#6. $\sin\alpha = 3/5$ $\quad$ $\sin\beta = \dfrac{24}{25}$



i) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\dfrac{3}{5}\right)\left(\dfrac{-7}{25}\right) + \left(\dfrac{4}{5}\right)\left(\dfrac{24}{25}\right)$

$\Rightarrow \dfrac{-21}{125} + \dfrac{96}{125} = \dfrac{75}{125}$

ii) $\cos 2\beta = \cos^2\beta - \sin^2\beta$

$= (-7/25)^2 - (24/25)^2 = \dfrac{49}{625} - \dfrac{576}{625} = \dfrac{-527}{625}$

7a) $\dfrac{\csc x - \sin x}{\cot^2 x} = \dfrac{\frac{1}{\sin x} - \frac{\sin x}{1}}{\frac{\cos^2 x}{\sin^2 x}} = \dfrac{\frac{1-\sin^2 x}{\sin x}}{\frac{\cos^2 x}{\sin^2 x}}$

$= \dfrac{1-\sin^2 x}{\sin x} \cdot \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{\cancel{\cos^2 x}}{\sin x} \cdot \dfrac{\sin^2 x}{\cancel{\cos^2 x}}$

$= \sin x$

7b) $\sin x + \cos x \cot x$

$= \sin x + \cos x \cdot \dfrac{\cos x}{\sin x}$

$= \dfrac{\sin x}{1} + \dfrac{\cos^2 x}{\sin x} = \dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x} = \dfrac{\sin^2 x + \cos^2 x}{\sin x} = \dfrac{1}{\sin x}$

7c) $\tan^2 x - \cos^2 x \cancel{\tan^2 x}$

$= \tan^2 x (1 - \cos^2 x)$

$= \tan^2 x \; (\sin^2 x)$

$= \dfrac{\sin^2 x}{\cos^2 x} \; \sin^2 x$

$= \dfrac{\sin^4 x}{\cos^2 x}$

#8. $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$\cos\frac{\pi}{2}\cos\theta - \sin\frac{\pi}{2}\sin\theta$

$(0)\cos\theta - (1)\sin\theta$

$-\sin\theta \quad | \quad -\sin\theta$

#9a) $\frac{1}{\sin x\cos x} - \frac{\cos x}{\sin x} = \tan x$

$\frac{1 - \cos x \cos x}{\sin x \cos x}$

$\frac{1-\cos^2 x}{\sin x \cos x}$

$\frac{\sin^2 x}{\sin x \cos x}$

$\frac{\sin x}{\cos x}$

$\tan x \quad | \quad \tan x$

9b) $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x}{1}$

$\frac{\sin^2 x - \sin^2 x\cos^2 x}{\cos^2 x}$

$\frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}$

$\frac{\sin^2 x(\sin^2 x)}{\cos^2 x}$

$\frac{\sin^2 x}{\cos^2 x} \cdot \sin^2 x$

$\tan^2 x \cdot \sin^2 x$

9c) $\frac{1-3\cos x-4\cos^2 x}{\sin^2 x} \quad | \quad \frac{1-4\cos x}{1-\cos x}$

$\frac{(1+\cos x)(1-4\cos x)}{1-\cos^2 x}$

$\frac{\cancel{(1+\cos x)}(1-4\cos x)}{(1-\cos x)\cancel{(1+\cos x)}}$

$\frac{1-4\cos x}{1-\cos x}$

9d) $\csc x \cos^2 x + \sin x = \csc x$

$$\frac{1}{\sin x} \cdot \cos^2 x + \frac{\sin x}{1}$$

$$\frac{\cos^2 x + \sin^2 x}{\sin x}$$

$$\frac{1}{\sin x}$$

$$\csc x$$

9e) $\sec^2 x + \tan^2 x \sec^2 x = \sec^4 x$

$$\sec^2 x(1 + \tan^2 x)$$

$$\sec^2 x \quad (\sec^2 x)$$

$$\sec^4 x$$

9f) $\dfrac{\sec x + \tan x}{\frac{1}{1-\sin x}} = \cos x$

$$\frac{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}{\frac{1}{1-\sin x}}$$

$$\left(\frac{1+\sin x}{\cos x}\right)\left(\frac{1-\sin x}{1}\right)$$

$$\frac{1-\sin^2 x}{\cos x}$$

$$\frac{\cos^2 x}{\cos x}$$

$$\cos x \quad | \quad \cos x$$

9g) $\dfrac{\cos x - \tan x}{\sin x \cos x} = \csc x - \sec^2 x$

$$\frac{\frac{\cos x}{1} - \frac{\sin x}{\cos x}}{\sin x \cos x}$$

$$\frac{\frac{\cos^2 x - \sin x}{\cos x}}{\sin x \cos x}$$

$$\frac{\cos^2 x - \sin x}{\cos x} \cdot \frac{1}{\sin x \cos x}$$

$$\frac{\cos^2 x - \sin x}{\sin x \cos^2 x}$$

$$\frac{\cancel{\cos^2 x}}{\sin x \cancel{\cos^2 x}} - \frac{\sin x}{\sin x \cos^2 x}$$

$$\frac{1}{\sin x} - \frac{1}{\cos^2 x}$$

$$= \csc x - \sec^2 x$$

9h) $(\cot A + \tan A)^2 = \csc^2 A \sec^2 A$

$(\cot A + \tan A)(\cot A + \tan A)$

$\cot^2 A + 2\cot A \tan A + \tan^2 A$

$\cot^2 A + \underbrace{2 + \tan^2 A}$

$\underbrace{\cot^2 A + 1} + \underbrace{1 + \tan^2 A}$

$\csc^2 A + (\sec^2 A)$

$\frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$

$\frac{\cos^2 A + \sin^2 A}{\sin^2 A \cos^2 A}$

$\frac{1}{\sin^2 A \cos^2 A}$

$= \frac{1}{\sin^2 A} \cdot \frac{1}{\cos^2 A}$

$= \sec^2 A \csc^2 A$

9i) $\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} = \csc x$

$\frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}} + \frac{1 - 2\sin^2 x}{\sin x}$

$2\sin x + \frac{1}{\sin x} - \frac{2\sin^2 x}{\sin x}$

$\cancel{2\sin x} + \frac{1}{\sin x} - \cancel{2\sin x}$

$\frac{1}{\sin x}$

$= \csc x$

9j) $\frac{1-\sin^2 x}{1+2\sin x-3\sin^2 x} = \frac{1+\sin x}{1+3\sin x}$

$\frac{\cancel{(1-\sin x)}(1+\sin x)}{\cancel{(1-\sin x)}(1+3\sin x)}$

$\frac{1+\sin x}{1+3\sin x}$

9k) $\frac{\sin x\cos x-\sin x}{\cos^2 x-1} = \frac{1-\cos x}{\sin x}$

$\frac{\sin x\cancel{(\cos x-1)}}{\cancel{(\cos x-1)}(\cos x+1)}$

$\frac{\sin x}{\cos x+1}$

$\frac{\sin x\cos x-\sin x}{\cos^2 x-1}$

$\frac{1-\cos x}{\sin x}\cdot\frac{\sin x}{\sin x}$

$\frac{\sin x-\sin x\cos x}{\sin^2 x}$

$\frac{\sin x-\sin x\cos x}{1-\cos^2 x}$ $\frac{(-1)}{(-1)}$

$\frac{\sin x\cos x-\sin x}{\cos^2 x-1}$

9l) $\frac{\cot x-\tan x}{\sin x\cos x} = \csc^2 x-\sec^2 x$

$\frac{\cot x}{\sin x\cos x} - \frac{\tan x}{\sin x\cos x}$

$\frac{\frac{\cos x}{\sin x}}{\sin x\cos x} - \frac{\frac{\sin x}{\cos x}}{\sin x\cos x}$

$\frac{\cancel{\cos x}}{\sin^2 x\cancel{\cos x}} - \frac{\sin x}{\sin x\cos^2 x}$

$\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}$

$\csc^2 x - \sec^2 x$

m) $2(\tan x + \cot x) = \dfrac{\sin 2x}{\cos^2 x \sin^2 x}$

$= 2\left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)$

$= 2 \dfrac{(\sin^2 x + \cos^2 x)}{\sin x \cos x}$

$= 2 \left(\dfrac{1}{\sin x \cos x}\right)$

$= \dfrac{2}{\sin x \cos x}$

$\dfrac{2 \sin x \cos x}{\cos^2 x \sin^2 x}$

$= 2 \dfrac{1}{\sin x} \cdot \dfrac{1}{\cos x}$

$= \dfrac{2}{\sin x \cos x}$

n) $\dfrac{\sin x}{1+\cos x} + \dfrac{\cos x}{\sin x}$ $\csc x$

$= \dfrac{\sin^2 x + \cos x(1+\cos x)}{(1+\cos x)(\sin x)}$

$= \dfrac{\sin^2 x + \cos x + \cos^2 x}{(1+\cos x)(\sin x)}$

$= \dfrac{\cancel{1+\cos x}}{\cancel{(1+\cos x)}(\sin x)}$

$= \dfrac{1}{\sin x}$

$= \csc x$

o) $\dfrac{\tan x + \sin x}{1+\cos x} = \tan x$

$\dfrac{\dfrac{\sin x}{\cos x} + \dfrac{\sin x}{1}}{1+\cos x}$

$= \dfrac{\dfrac{\sin x + \sin x \cos x}{\cos x}}{1+\cos x}$

$= \dfrac{\sin x(\cancel{1+\cos x})}{\cos x} \cdot \left(\dfrac{1}{\cancel{1+\cos x}}\right)$

$= \dfrac{\sin x}{\cos x}$

$= \tan x$

#10 a) $\sin 2x = \sin x$ $\quad 0 \le x \le 2\pi$

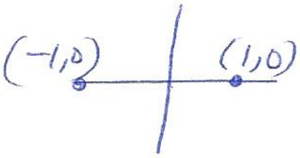
$2\sin x \cos x = \sin x$

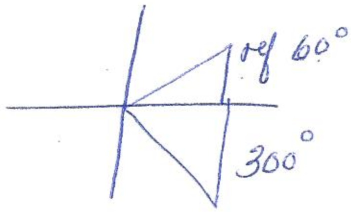
$2\sin x \cos x - \sin x = 0$

$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0 \qquad \cos x = \frac{1}{2}$

$\boxed{x = 0,\ \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}}$





10 b) $\tan x \cos x \sin x - 1 = 0$ $\quad 0° \le x \le 360°$

Restrictions:

$\cos x \ne 0$

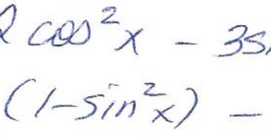
$x \ne 90°, 270°$

$\frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}\, \sin x - 1 = 0$

$\sin^2 x - 1 = 0$

$\sin^2 x = 1$

$\sin x = \pm 1$



$x = \cancel{90°, 270°}$

$\boxed{\text{No Solutions}}$

10c) $2\cos^2 x - 3\sin x - 3 = 0$

$2(1 - \sin^2 x) - 3\sin x - 3 = 0$

$2 - 2\sin^2 x - 3\sin x - 3 = 0$

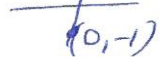
$-2\sin^2 x - 3\sin x - 1 = 0$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2} \qquad \sin x = -1$

ref 30°



$x = 210° + 360°k \quad k \in I$

$x = 330° + 360°k \quad k \in I$

$x = \frac{3\pi}{2} + 360°k \quad k \in I$

10d) $\cos 2x + 3\cos x + 2 = 0$

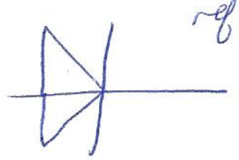
$2\cos^2 x - 1 + 3\cos x + 2 = 0$

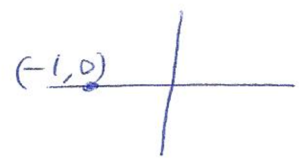
$2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = -1$

ref 60°





$x = 120^\circ + 360^\circ k \quad k \in I$ $\qquad$ $x = 180^\circ + 360^\circ k \quad k \in I$

$x = 240^\circ + 360^\circ k \quad k \in I$

10e) $3\csc x - \sin x = 2 \quad 0 \le x \le 2\pi$

$\left[3 \frac{1}{\sin x} - \sin x = 2\right](x \sin x)$ $\qquad$ $\sin x \neq 0$

$x \neq 0, \pi, 2\pi$

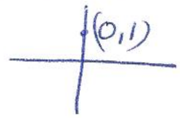
$3 - \sin^2 x = 2\sin x$

$0 = \sin^2 x + 2\sin x - 3$

$0 = (\sin x + 3)(\sin x - 1)$

$\sin x = -3$ $\qquad$ $\sin x = 1$

reject



$\boxed{x = 90^\circ = \pi/2}$

10f) $\cos x \tan x - \sin^2 x = 0$

$\cos x \cdot \frac{\sin x}{\cos x} - \sin^2 x = 0$ $\qquad$ $\cos x \neq 0$

$x \neq 90^\circ, 270^\circ$

$\sin x - \sin^2 x = 0$

$\sin x (1 - \sin x) = 0$

$\sin x = 0$ $\qquad$ $\sin x = 1$

$\boxed{x = 0^\circ, 180^\circ}$ $\qquad$ $x = \cancel{90^\circ}$