Mathematics 3200 Unit: Trigonometric Identities

Name:_____

1. Calculate the exact value of each expression.

a) $\sin 75^0 \cos 15^0 + \cos 75^0 \sin 15^0$

b) $\cos\frac{5\pi}{12}\cos\frac{\pi}{12} - \sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

c) $\sin\frac{11\pi}{12}$

d) $\dfrac{\tan\frac{2\pi}{3} + \tan\frac{\pi}{12}}{1 - \tan\frac{2\pi}{3}\tan\frac{\pi}{12}}$

2. Write $2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$ as a single trigonometric function.

3. What are the non-permissible values for x for the equation $\csc x \cos x = \cot x$?

4. What is the exact value of $\cot(-165^0)$?

5. What is the exact value of $\cos 75^0$?

6. If $\sin\alpha = \frac{3}{5}$ and $\sin\beta = \frac{24}{25}$, where $0 < \alpha < \frac{\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$

(i) determine the value of $\sin(\alpha + \beta)$

(ii) determine the value of $\cos 2\beta$

7. Simplify

a) $\dfrac{\csc x - \sin x}{\cot^2 x}$

b) $\sin x + \cos x \cot x$

c) $\tan^2 x - \cos^2 x \tan^2 x$

8. Prove $\cos(\frac{\pi}{2} + \theta) = -\sin\theta$

9. Prove the following:

a) $\dfrac{1}{\sin x \cos x} - \dfrac{\cos x}{\sin x} = \tan x$

(b) $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

(c) $\dfrac{1-3\cos x - 4\cos^2 x}{\sin^2 x} = \dfrac{1-4\cos x}{1-\cos x}$

(d) $\csc x \cos^2 x + \sin x = \csc x$

(e) $\sec^2 x + \tan^2 x \sec^2 x = \sec^4 x$

(f) $\dfrac{\sec x + \tan x}{\dfrac{1}{1-\sin x}} = \cos x$

(g) $\dfrac{\cos x - \tan x}{\sin x \cos x} = \csc x - \sec^2 x$

(h) $(\cot A + \tan A)^2 = \csc^2 A \sec^2 A$

(i) $\dfrac{\sin 2x}{\cos x} + \dfrac{\cos 2x}{\sin x} = \csc x$

(j) $\dfrac{1-\sin^2 x}{1+2\sin x - 3\sin^2 x} = \dfrac{1+\sin x}{1+3\sin x}$

(k) $\dfrac{\sin x \cos x - \sin x}{\cos^2 x - 1} = \dfrac{1-\cos x}{\sin x}$

(l) $\dfrac{\cot x - \tan x}{\sin x \cos x} = \csc^2 x - \sec^2 x$

(j) $2(\tan x + \cot x) = \dfrac{\sin 2x}{\cos^2 x \sin^2 x}$

(k) $\dfrac{\sin x}{1+\cos x} + \dfrac{\cos x}{\sin x} = \csc x$

(l) $\dfrac{\tan x + \sin x}{1+\cos x} = \tan x$

10. Solve the following:

(a) $\sin 2x = \sin x$ where $0 \le x \le 2\pi$

(b) $\tan x \cos x \sin x - 1 = 0$ where $0° \le x \le 360°$

(c) $2\cos^2 x - 3\sin x - 3 = 0$ general solution in degrees

(d) $\cos 2x + 3\cos x + 2 = 0$ general solution in degrees

(e) $3\csc x - \sin x = 2$ where $0 \le x \le 2\pi$

(f) $\cos x \tan x - \sin^2 x = 0$ where $0° \le x \le 360°$