

Math 3200: ch. 4 Review - Trigonometry and the Unit Circle ①

1. $160 \times \frac{\pi}{180} = \frac{8\pi}{9}$ (C)

2. $8 \text{ rad} \times \frac{180}{\pi} = 458.6$ (B)

$$458.6 - 360 = 98.6$$

$\therefore 8 \text{ rad}$ in Q2

3. $\cot(200^\circ) + \csc(\beta)$ (C)

cal in degree $\rightarrow \frac{1}{\tan 200^\circ} + \frac{1}{\sin(\beta)}$ ← cal in radians.
 $2.75 + 7.09$
 $= 9.84$

4. $\cot \theta < 0$, $\sec > 0$ (D)

$$\frac{1}{\tan \theta} < 0 \quad \frac{1}{\cos} > 0$$

neg pos

S/A
T/C

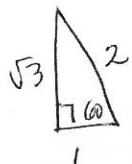
Quadrant (4)

5. $\csc x - 2 = 0$ (D)

$$\csc x = 2$$

$$\frac{1}{\sin x} = 2$$

$$\sin x = \frac{1}{2}$$



$$\text{ref } \angle = 30^\circ$$

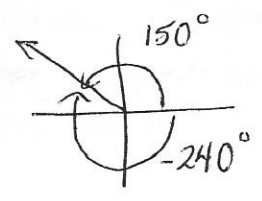
sin pos in Q1 + Q2

$$\therefore \theta = 30^\circ + \theta = 180 - 30 = 150$$
$$= \frac{5\pi}{6}$$

10. (C)

$$\frac{5\pi}{6} + \frac{-7\pi}{6}$$

$$150^\circ + -210^\circ$$



$$150^\circ - 360^\circ = -240^\circ$$

11. $\tan 30^\circ + \cot (30^\circ)$

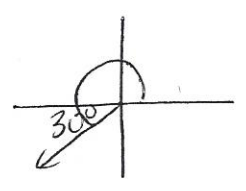
$$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + \sqrt{3}$$

$$= \frac{\sqrt{3}}{3} + \frac{3\sqrt{3}}{3}$$

$$= \frac{4\sqrt{3}}{3} \quad (B)$$



12. $\frac{7\pi}{6} = 7(30^\circ) = 210^\circ \quad (D)$



13. (C)

$$\left(\frac{3}{5}, -\frac{4}{5}\right)$$

x y

$$x^2 + y^2 = 1$$

$$\left(\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2 = 1$$

$$\frac{9}{25} + \frac{16}{25} = 1$$

$$\frac{25}{25} = 1$$

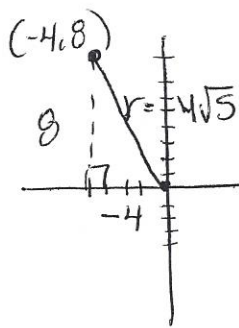
1. C
2. B
3. C
4. D
5. D

6. A
7. B
8. D
9. D
10. C

11. B
12. D
13. C

(4)

14. (-4, 8)



$$r^2 = (8)^2 + (-4)^2$$

$$r^2 = 64 + 16$$

$$r^2 = 80$$

$$r = \pm\sqrt{80}$$

$$r = 4\sqrt{5}$$

ref \angle : $\theta = \tan^{-1}(2)$
 $= 63.4^\circ$
 $\theta = 180^\circ - 63.4^\circ = 116.6^\circ$

$$\sin \theta = \frac{8}{4\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{-4}{4\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\tan \theta = \frac{8}{-4} = -2$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$$

$$\sec \theta = \frac{1}{\cos \theta} = -\sqrt{5}$$

$$\cot \theta = \frac{1}{\tan \theta} = -\frac{1}{2}$$

15. a) $\sin\left(-\frac{4\pi}{3}\right) + \sec\left(\frac{\pi}{4}\right)$

$$= -\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{1}$$

$$= \frac{-\sqrt{3} + 2\sqrt{2}}{2}$$

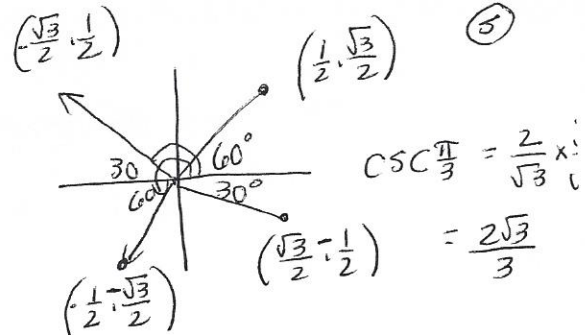
$$15. b) \frac{\cos\left(\frac{5\pi}{6}\right) + \sin(240^\circ)}{\csc\left(\frac{\pi}{3}\right) \sin\left(\frac{11\pi}{6}\right)}$$

$$= \frac{-\frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{2}\right)}{\frac{1}{\sin 60} \cdot \left(-\frac{1}{2}\right)}$$

$$= \frac{-\frac{2\sqrt{3}}{2}}{-\frac{\sqrt{3}}{3}}$$

$$= -\sqrt{3} \times \frac{-3}{\sqrt{3}}$$

$$= 3$$



$$16. a) 6\tan^2(x) - \tan(x) - 15 = 0$$

$$(2\tan\theta + 3)(3\tan\theta - 5) = 0$$

$$2\tan\theta + 3 = 0 \quad 3\tan\theta - 5 = 0$$

$$\tan\theta = -\frac{3}{2} \quad \tan\theta = \frac{5}{3}$$

$$\text{ref: } \theta = \tan^{-1}\left(\frac{3}{2}\right)$$

$$= 56^\circ$$

\tan is neg in:

$$Q2: \theta = 180 - 56 = 124^\circ$$

$$Q4: \theta = 360 - 56 = 304^\circ$$

$$\text{ref: } \theta = \tan^{-1}\left(\frac{5}{3}\right)$$

$$\theta = 59^\circ$$

\tan is pos in:

$$Q1: \theta = 59^\circ$$

$$Q3: \theta = 180 + 59 = 239^\circ$$

$$\theta = \begin{cases} 124^\circ + 360^\circ k, k \in \mathbb{I} \\ 304^\circ + 360^\circ k, k \in \mathbb{I} \\ 59^\circ + 360^\circ k, k \in \mathbb{I} \\ 239^\circ + 360^\circ k, k \in \mathbb{I} \end{cases}$$

16. b) $2\sin^2 x + 5\sin x + 3 = 0$

$$(2\sin x + 3)(\sin x + 1) = 0$$

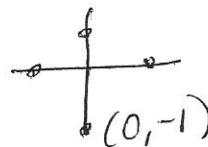
$$2\sin x + 3 = 0 \quad \sin x + 1 = 0$$

$$\sin x = -\frac{3}{2} \quad \sin x = -1$$

$$\text{ref } \angle: x = \sin^{-1}\left(\frac{3}{2}\right) \quad x = 270^\circ$$

no solution

$$x = 270^\circ + 360K, K \in \mathbb{I}$$



c) $\sec^2 x - 2\sec x - 3 = 0$

$$(\sec x - 3)(\sec x + 1) = 0$$

$$\sec x - 3 = 0 \quad \sec x + 1 = 0$$

$$\sec x = 3 \quad \sec x = -1$$

$$\frac{1}{\cos x} = 3$$

$$\frac{1}{\cos x} = -1$$

$$\cos x = \frac{1}{3}$$

$$\cos x = -1$$

$$x = \cos^{-1}\left(\frac{1}{3}\right)$$

$$x = \pi$$

$$x = 1.23 \text{ rad}$$

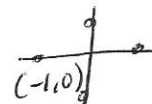
$\frac{S}{T} \frac{A}{C}$

cos is pos. in:

$$Q1: x = 1.23$$

$$Q4: x = 2\pi - 1.23 = 5.05$$

$$x = \begin{cases} \pi + 2\pi K, K \in \mathbb{I} \\ 1.23 + 2\pi K, K \in \mathbb{I} \\ 5.05 + 2\pi K, K \in \mathbb{I} \end{cases}$$



d) $(\tan x - 1)(\tan x - \sqrt{3}) = 0$

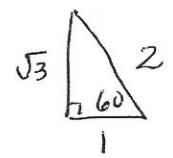
$\tan x - 1 = 0$ $\tan x - \sqrt{3} = 0$

$\tan x = 1$

$\tan x = \sqrt{3}$

$x = \frac{\pi}{4}$

$x = \frac{\pi}{3}$



tan is pos in

tan is pos in:

Q1: $x = \pi/4$

Q1: $x = \pi/3$

Q3: $x = \frac{5\pi}{4}$

Q3: $x = \pi + \pi/3 = 4\pi/3$

S/A
T/C

$x = \begin{cases} \frac{\pi}{4} + 2\pi k, k \in \mathbb{I} \\ \frac{5\pi}{4} + 2\pi k, k \in \mathbb{I} \\ \frac{\pi}{3} + 2\pi k, k \in \mathbb{I} \\ \frac{4\pi}{3} + 2\pi k, k \in \mathbb{I} \end{cases}$

OR $x = \begin{cases} \frac{\pi}{4} + \pi k, k \in \mathbb{I} \\ \frac{\pi}{3} + \pi k, k \in \mathbb{I} \end{cases}$

17. a) $\sec^2 x = 3\sec x - 2$ $-\pi \leq x < 2\pi$

$\sec^2 x - 3\sec x + 2 = 0$

$(\sec x - 1)(\sec x - 2) = 0$

$\sec x = 1$

$\sec x = 2$

$\frac{1}{\cos x} = 1$

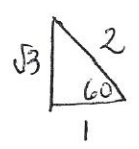
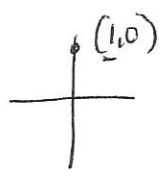
$\frac{1}{\cos x} = 2$

$\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{2}$

$x = \frac{\pi}{3}$



cos is pos in

Q1: $x = \pi/3$

Q4: $x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ + $x = \frac{5\pi}{3} - 2\pi = -\frac{\pi}{3}$

$x = \frac{\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}$

b) $\sec^2 \theta - 4 = 0$

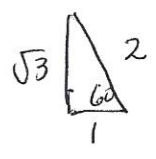
OR $[-180, 180]$
 $(\sec \theta - 2)(\sec \theta + 2) = 0$

$\sec^2 \theta = 4$

$\sec \theta = \pm 2$

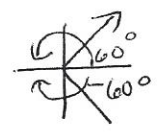
$\frac{1}{\cos \theta} = \pm 2$

$\cos \theta = \pm \frac{1}{2}$



S/A
T/C

$\theta = 60^\circ$



cos pos. in Q1 + Q4

$\theta = 60^\circ, -60^\circ$

c) $\cot^2 \theta = 4 \cot \theta$ $[0, 2\pi]$

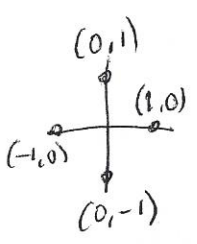
$\cot^2 \theta - 4 \cot \theta = 0$

$\cot \theta (\cot \theta - 4) = 0$

$\cot \theta = 0$ $\cot \theta = 4$

$\frac{1}{\tan \theta} = 0$ $\frac{1}{\tan \theta} = 4$

$\tan \theta = 0$ $\tan \theta = \frac{1}{4}$



$\frac{\sin \theta}{\cos \theta} = \frac{y}{x} = 0$

$\theta = \tan^{-1}(\frac{1}{4})$

$\theta = 0, \pi$

$\theta = 0.25$

tan is pos in

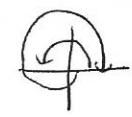
Q1: $\theta = 0.25$

Q3: $\theta = \pi + 0.25 = 3.38$

$\theta = 0, 0.25, \pi, 3.38$

d)
$$\begin{aligned} 3\csc x - 5 &= 4\csc x - 7 \\ -3\csc x + 7 & \quad -3\csc x + 7 \end{aligned}$$

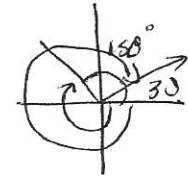
$$[-360, 180^\circ]$$



$$\csc x = 2$$

$$\frac{1}{\sin x} = 2$$

$$\sin x = \frac{1}{2}$$

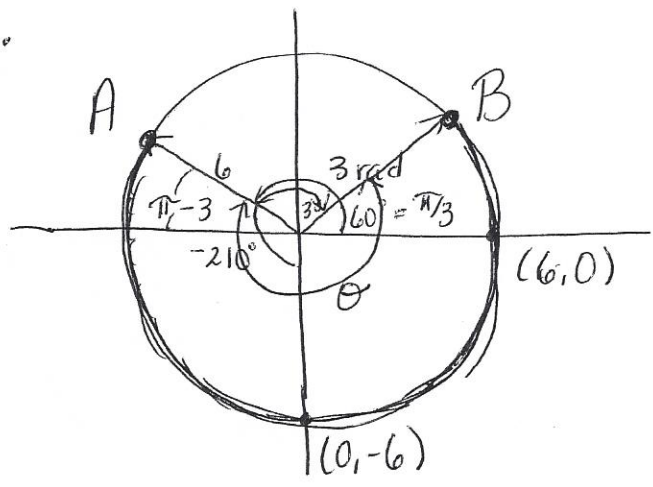


$$x = 30^\circ$$

sin is pos in Q1 + Q2

$$x = 30^\circ, 150^\circ, -210^\circ, -330^\circ$$

18.



$$\begin{aligned} \theta &= \frac{\pi}{3} + \pi + (\pi - 3) \\ &= \frac{\pi}{3} + 2\pi - 3 \\ &= \frac{\pi}{3} + \frac{6\pi}{3} - 3 \\ &= \left(\frac{7\pi}{3} - 3\right) \end{aligned}$$

$$\theta = 4.33$$

Exact:

$$\begin{aligned} a &= r \cdot \theta \text{ (in radians)} \\ &= 6 \cdot \left(\frac{7\pi}{3} - 3\right) \\ &= \frac{42\pi}{3} - 18 \end{aligned}$$

$$\begin{aligned} a &= r \cdot \theta \\ &= 6 \times 4.33 \\ &= 25.98 \text{ cm} \end{aligned}$$