Section 9.3 Binomial Theorem

Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Blaise Pascal, a french mathematician, described a triangular array of numbers that represent the coefficients in a binomial expansion $(x+y)^n, n \varepsilon N$

$$(x+y)^0 =$$

$$(x+y)^1 =$$

$$(x+y)^2 =$$

$$(x+y)^3 =$$

$$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$

Binomial	Pascal's Triangle in Binomial Expansion	Row
$(x+y)^0$	1	1
$(x+y)^1$	$1x + 1y$	2
$(x+y)^2$	$1x^2 + 2xy + 1y^2$	3
$(x+y)^3$	$1x^3 + 3x^2y + 3xy^2 + 1y^3$	4
$(x+y)^4$	$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$	5

powers of x decrease from 4 to 0
powers of y increase from 0 to 4

Note: The coefficients for the terms of the expansion of $(x+y)^6$ occur in the 7th row. How many terms?

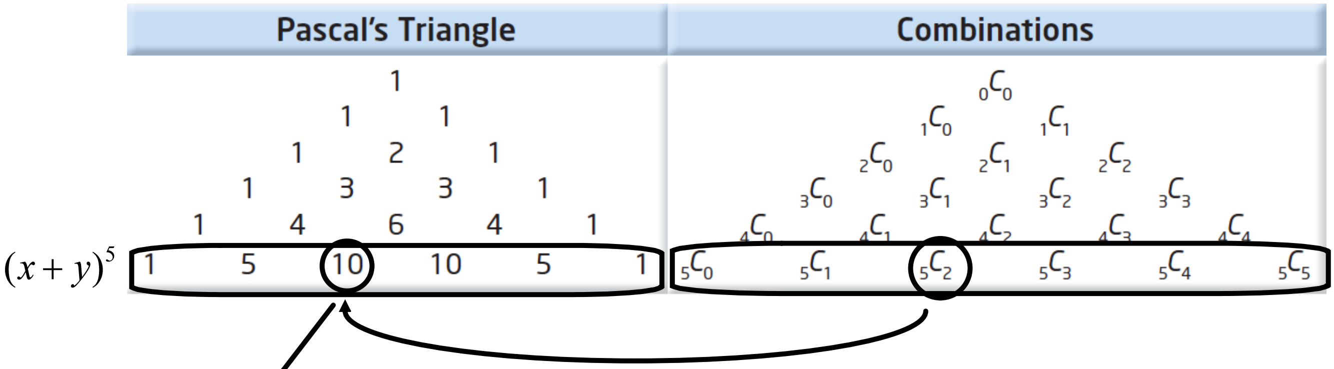
Example 1: The coefficients in Pascal's triangle for Row 5 is:

1 4 6 4 1

Expand $(2t-3w)^4$

Think about $x = 2t$ and $y = -3w$

The coefficients in a binomial expansion can also be determined using combinations.



$$
\begin{aligned}
{}_5C_2 &= \frac{5!}{3!2!} \\
&= \frac{(5)(4)}{2} \\
&= 10
\end{aligned}
$$

Note that ${}_5C_2$ represents the number of combinations of five items taken two at a time. In the expansion of $(x + y)^5$, it represents the coefficient of the term containing x^3y^2 and shows the number of selections possible for three *x*'s and two *y*'s.

Example 2: Expand $(x+y)^5$

Method 1: Use Row 6 of Pascal's Triangle. What are the coefficients?

Method 2: Use Combinations to determine the coefficients
(Binomial Theorem)

$(x+y)^5 =$

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You can use the **binomial theorem** to expand any power of a binomial expression.

$$(x+y)^n = {}_nC_0(x)^n(y)^0 + {}_nC_1(x)^{n-1}(y)^1 + {}_nC_2(x)^{n-2}(y)^2 + \cdots + {}_nC_{n-1}(x)^1(y)^{n-1} + {}_nC_n(x)^0(y)^n$$

Example 3: Use the binomial theorem to expand $(2a-3b)^4$

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You can use the **binomial theorem** to expand any power of a binomial expression.

t_2 t_3

$$(x+y)^n = {}_nC_0(x)^n(y)^0 + {}_nC_1(x)^{n-1}(y)^1 + {}_nC_2(x)^{n-2}(y)^2 + \cdots + {}_nC_{n-1}(x)^1(y)^{n-1} + {}_nC_n(x)^0(y)^n$$

general term $t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$

Example 4: What is the third term in the expansion $(4b-5)^6$?

$t_{k+1} = {}_nC_k(x)^{n-k}(y)^k$ $x=$ $n=$

$t_3=$ $y=$ $k=$

→

Example 5: Expand $\left(2a^2 - \frac{3}{y}\right)^4$

Example 6: What is the fourth term in the expansion of $\left(2x^3 - 3y^2\right)^7$?

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Example 7:

Given $(5x-2y)^9$, determine the coeficient of the term containing x^5.

Example 8: Given $(3x-4)^8$, determine the midle term of the expansion.

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Example 9:

A term in the expansion of $(x+a)^7$ is $\frac{2150x^5}{y^4}$. Find the value of a .

p.542 #4 ab, 5b, 6ab, 7abc, 17a, 18