CHAPTER 9

(Chapter 11 in Resource)

Permutations, Combinations and the Binomial Theorem

How many ways can items be arranged?

- Fundamental Counting Principle
- Factorial
- Permutation
- Combination

} Counting Methods

Factorial

→ multiply consecutive numbers decreasing by 1.

$7\times6\times5\times4\times3\times2\times1$

Permutation $({}_nP_r)$

→ order matters (selection of objects)

Examples:
- password or code
- selecting a group to be president, vice-president, treasurer
- awarding medals to 1st place, 2nd place, 3rd place

Combination $({}_nC_r)$

→ order does not matter (selection of objects)

Examples:
- lottery
- pick a team of 5 people from a group of 10

Think About:

(1) How many ways can you arrange the letters in the word MICRO? no repetition

(2) How many ways can you arrange the letters in the word POSSIBILITY? repetition

(3) How many three letter words can be made from the letters of the word KEYBOARD? arranging a subset of items

(4) How many ways can Beth, John, Doug, and Mary be seated in a row if Doug must be in the second seat? specific position

(5) How many arrangements of the word ACTIVE are there if C and E must always be together? items together

→

Fundamental Counting Principle

→ determine the total number of possible outcomes that can occur with a group(s).

Example 1

The school cafeteria advertises that it can serve up to 24 different meals consisting of one item from each of the three categories:

Fruit:	Apples (A), Bananas(B) or Cantaloupe(C)
Sandwiches:	Roast Beef (R) or Turkey (T)
Beverages:	Lemonade (L), Milk (M), Orange Juice (O) or Pineapple Juice (P)

Is their advertising correct?

Fundamental Counting Principle:

____ choices for fruit   ____ choices for sandwich   ____ choices for beverage   ★

If one task can be performed in a ways,

a second task can be performed in b ways,

and a third task be performed in c ways, then the number of ways to perform all the tasks together is abc

Distinguish between the words AND/OR

ways to choose a fruit and a sandwich and a beverage,

→ (multiply individual selections)

3 fruit choices x 2 sandwich choices x 4 beverage choices

= 24 possibilities

ways to choose a fruit or a sandwich or a beverage

→ (add individual selections)

3 fruit choices + 2 sandwich choices + 4 beverage choices

= 9 possibilities

Fundamental Counting Principle

Arrangements Without Restrictions

Example 2

A store manager has selected 4 possible applicants for two different positions at a department store. In how many ways can the manager fill the positions?

___ # of choices for position 1 and ___ # of choices for position 2

\# of ways to fill the positions ___

Example 3

How many ways can the letters in the word PENCIL be arranged?

Idea: We have 6 objects and 6 possible positions to occupy

___ ___ ___ ___ ___ ___

Factorial

Many examples involve arrangements where you multiply consecutive numbers decreasing by 1.

Previous example: $6\times5\times4\times3\times2\times1$

This product can be written in compact form as $6!$

General: $n! = n(n-1)(n-2)(n-3)........(2)(1)$ where $n \in N$

$0! = 1$

Example 4: Simplify factorial expressions

a) $\dfrac{100!}{97!}$

b) $\dfrac{n!}{(n-2)!}$

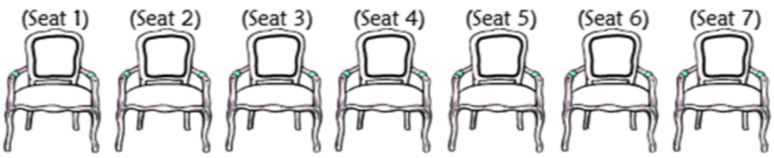
c) $\dfrac{3!(n+1)!}{2!(n-2)!}$

Fundamental Counting Principle

Arrangements With Restrictions

Example 5

In how many ways can a teacher seat 4 boys and three girls in a row of 7 seats if a boy must be seated at each of the row?



Restriction: a boy must be in each end seat.

- Fill seats 1 and 7 first
- Then fill remaining seats

Example 6

A 5-digit password is to be created using the digits 0-9.

a) How many arrangements are possible if repetition is allowed?

b) How many arrangements are possible if repetition is not allowed?

c) How many arrangements are possible if consecutive digits are not allowed?

Permutation

↳ ***arranging objects where order is important***

Formula to determine the number of permutations of n different elements taken r at a time

$$_nP_r = \frac{n!}{(n-r)!}$$

- arranging a subset of items
- only some of the items are used in the arrangement

Example 7

How many ways are there to arrange 3 people of a group of 5 in a line?

____ ____ ____

Fundamental Counting Principle:

Permutation Formula:

Example 8

In how many ways can 6 people be arranged in a line for a photograph?

Example 9

How many 4 letter words can be created if repetitions are not allowed?

→

Example 10

If there are 7 members on the student council, how many ways can the council select 3 students to be the president, vice-president and the treasurer?

Example 11

How many three letter words can be made from the letters of the word KEYBOARD?

Example 12

A code consists of three letters chosen from A to Z and three digits chosen from 0 to 9, with no repetitions of letters or numbers. Determine the total number of possible codes.

Example 13: Solve equations using ${}_{n}P_{r} = k$ where $n > 0, n > r$

a) Solve ${}_{n}P_{2} = 30$

b) Solve ${}_{n-1}P_{2} = 12$

c) Solve ${}_{5}P_{r} = 5!$ ★

Example 14:

Show that $100! + 99! = 101(99!)$ without technology.

→

Example 15

If there are 56 games in a series, and each team played every other team twice, once at home and once away, how many teams are there?

→

Permutations with Repeating Objects

In some cases, some of the items we want to arrange are identical.

If a set of n objects , with a of one kind that are identical, b of a second kind..... the entire set of objects can be arranged in

$$\frac{n!}{a!b!c!}$$

Example 16

How many different ways can you arrange the letters in the word POOL?

Example 17

How many different ways can you arrange the letters in the word MATHEMATICS?

Permutations

↳ *With Constraints*

- two or more objects must be placed together
- two or more objects cannot be placed together
- certain objects must be placed in certain positions

Example 18

How many ways can a group of 5 people be arranged in a line if two of them are good friends and want to sit together.

★ when certain items are to be kept together, treat the joined item as if they were only one object.

| Mary Beth |

| Dave | | Kyle |

| John |

→

Example 19

How many ways can a group of 5 people be arranged in a line if Mary and Beth should <u>not</u> sit together.

Complement

total # of arrangements with no restrictions — arrangements with Mary and Beth together

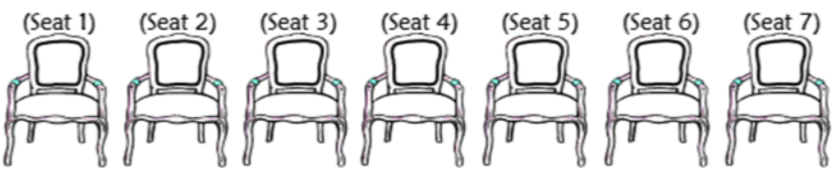
Arrangements involving Cases

Some problems have more than one case. Calculate the number of arrangements for each case and then add up the values for all cases to obtain the total.

Key Words: At Least, At Most , Either

Example 20

Determine the number of arrangements of 4 girls and 3 boys in a row of seven seats if the ends of the rows must be either both female or both male.



4 girls, 3 boys

Case for Females	Case for Males