Lesson 5.2: Transformations of Sinusoidal Functions (Sine and Cosine)

→ Reflections

Horizontal Translation (c)

Vertical Translation (d)

$$y = a\sin b(x-c)+d$$
$$y = a\cos b(x-c)+d$$

Remember:

vertical stretch → $|a|$

horizontal stretch → $\frac{1}{|b|}$

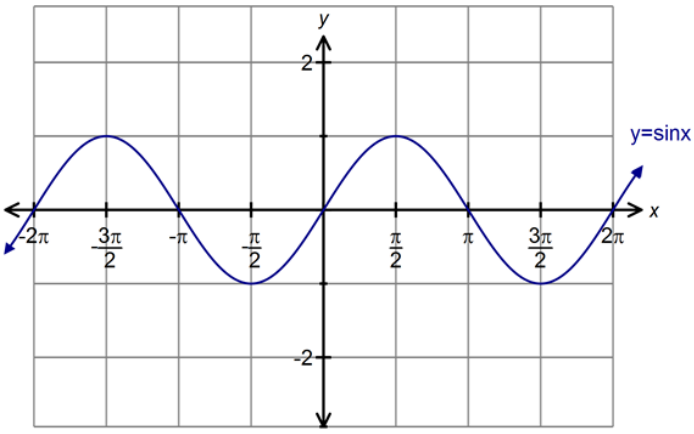
Part A: Reflections on the x and y-axis

Example 1: Graph the functions

$y = \sin x$

$y = -\sin x \longrightarrow$

$y = \sin(-x) \longrightarrow$



$y = \sin x$ $\qquad$ $y = -\sin(x)$

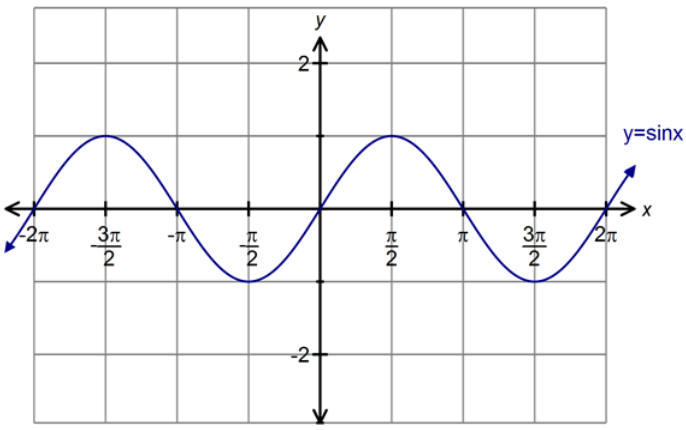
$(0,0) \longrightarrow$

$(\frac{\pi}{2},1) \longrightarrow$

$(\pi,0) \longrightarrow$

$(\frac{3\pi}{2},-1) \longrightarrow$

$(2\pi,0) \longrightarrow$



$y = \sin x$ $\qquad$ $y = \sin(-x)$

$(0,0) \longrightarrow$

$(\frac{\pi}{2},1) \longrightarrow$

$(\pi,0) \longrightarrow$

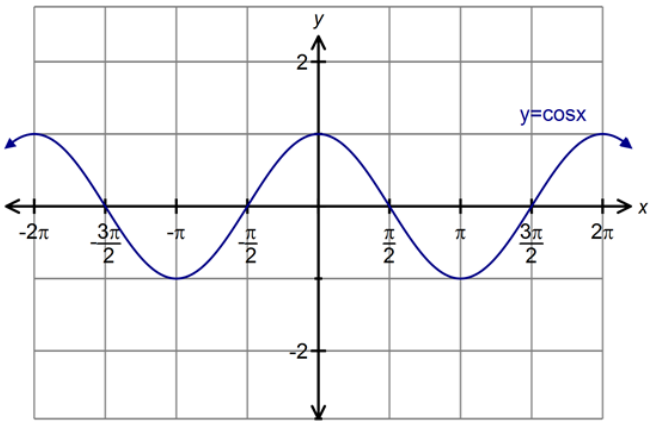
$(\frac{3\pi}{2},-1) \longrightarrow$

$\longrightarrow$

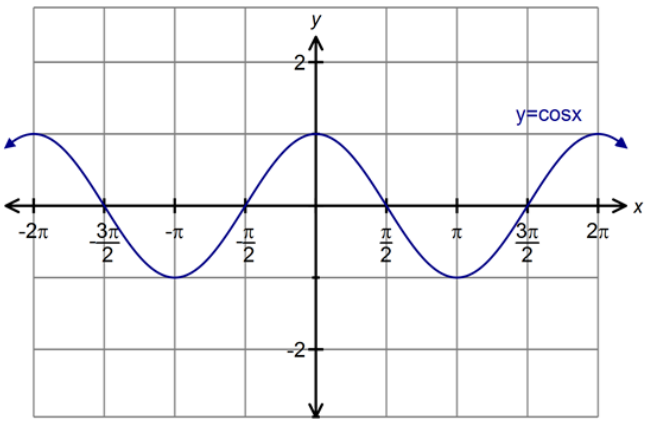
Example 2: Graph the functions $y = \cos x$

$y = -\cos x \longrightarrow$

$y = \cos(-x) \longrightarrow$



$y = \cos x$	$y = -\cos(x)$
$(0,1)$ $\longrightarrow$	
$(\frac{\pi}{2},0)$ $\longrightarrow$	
$(\pi,-1)$ $\longrightarrow$	
$(\frac{3\pi}{2},0)$ $\longrightarrow$	
$(2\pi,1)$ $\longrightarrow$	



$y = \cos x$	$y = \cos(-x)$
$(0,1)$ $\longrightarrow$	
$(\frac{\pi}{2},0)$ $\longrightarrow$	
$(\pi,-1)$ $\longrightarrow$	
$(\frac{3\pi}{2},0)$ $\longrightarrow$	

$\longrightarrow$

Part B: Vertical Translation d ⟶ (vertical displacement)

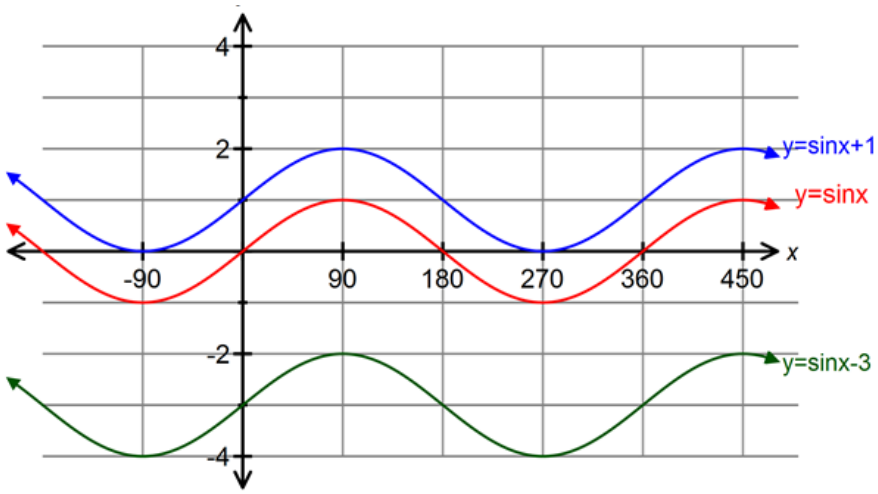
Example 3: Graph the functions

$y = \sin x$

$y = \sin x + 1$

$y = \sin x - 3$

How does changing the value of d affect the graph?



$y = \sin x$	$y = \sin x + 1$	$y = \sin x - 3$
vertical translation:	vertical translation:	vertical translation:
midline:	midline:	midline:
maximum:	maximum:	maximum:
minimum:	minimum:	minimum:

☆ Use amplitude and vertical translation to determine the maximum and minimum value

⟶

Part C: Horizontal Translation c ⟶ *(Phase Shift)*

- Cosine Function ⟶ the start of the first cycle of the cosine curve (0,1)
- Sine Function ⟶ the start of the first cycle of the sine curve (0,0)

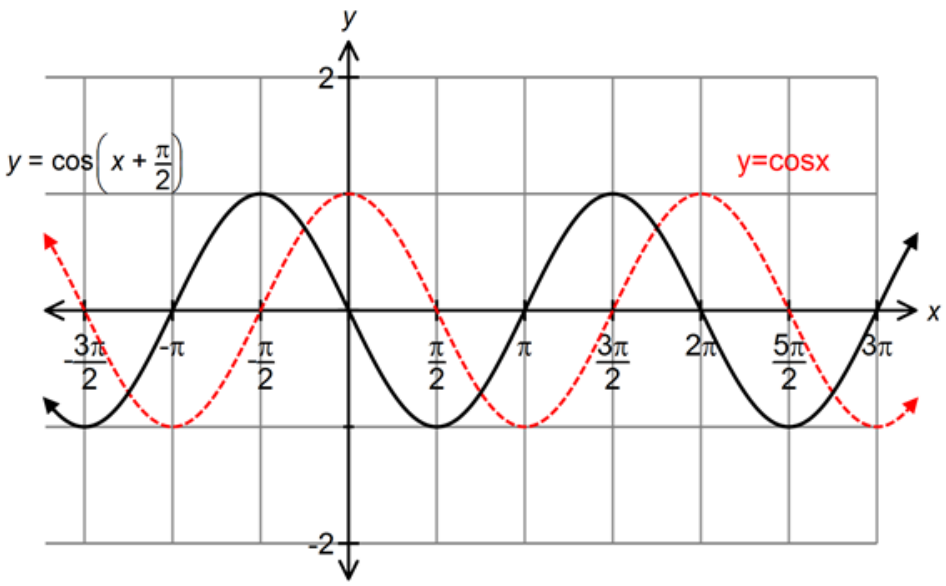
Example 4:

Graph the functions

$y = \cos x$

$y = \cos(x + \frac{\pi}{2})$

How does changing the value of c affect the graph?



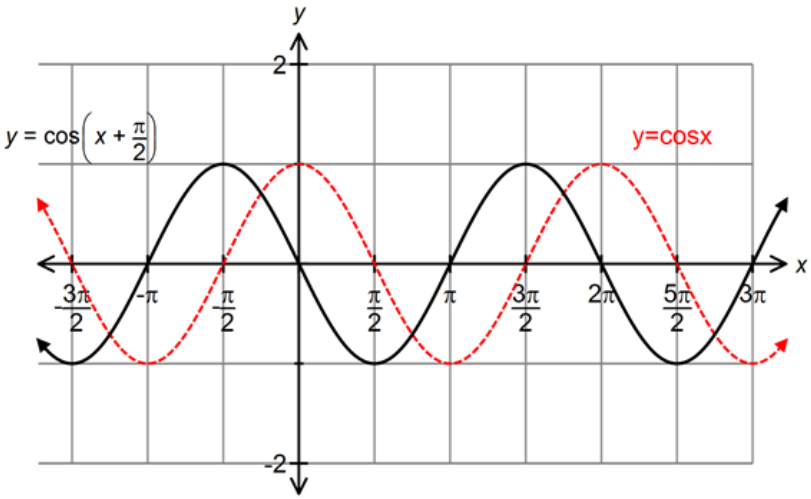
Locate the start of the first cycle of the cosine curve (0,1)

$y = \cos x$

$y = \cos(x + \frac{\pi}{2})$

→

Since the function is periodic, there are several equations that can correspond to a given graph where the phase shift is different.



Think about the equations:

$y = \cos(x - \frac{3\pi}{2})$ $\qquad$ $y = -\cos(x - \frac{\pi}{2})$ $\qquad$ $y = \cos(-(x + \frac{\pi}{2}))$

The value that is chosen for the phase shift will determine whether the graph is perceived as having a reflection on the x-axis or not.

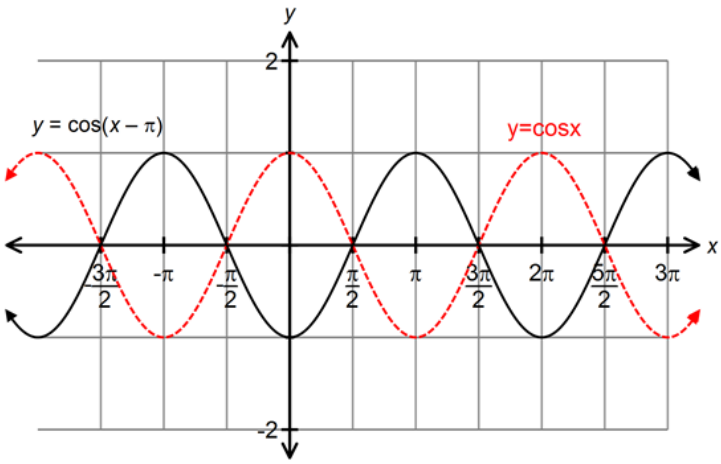
The x-value of the maximum point $\longrightarrow$ no reflection

The x-value of the minimum point $\longrightarrow$ reflection on x-axis

Example 5: Graph the functions

$$y = \cos x$$

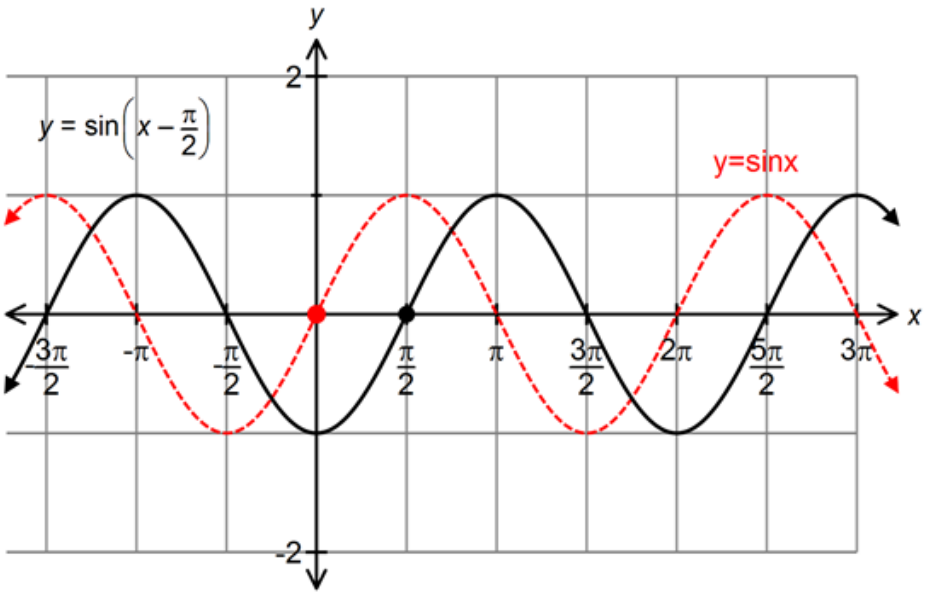
$$y = \cos(x - \pi)$$



Can you identify other equations that would produce the graph of $y = \cos(x - \pi)$?

Example 6: Graph the functions

$$y = \sin x$$
$$y = \sin(x - \tfrac{\pi}{2})$$



Locate the start of the first cycle of the sine curve (0,0)

Think about other equations:

$y = \sin(x - \frac{5\pi}{2})$ $\qquad$ $y = \sin(x + \frac{3\pi}{2})$ $\qquad$ $y = -\sin(x - \frac{3\pi}{2})$

The value that is chosen for the phase shift will determine whether the graph is perceived as having a reflection on the x-axis or not.

(i) where the x-coordinate hits the sinusoidal axis going from a minimum to a maximum $\longrightarrow$ no reflection

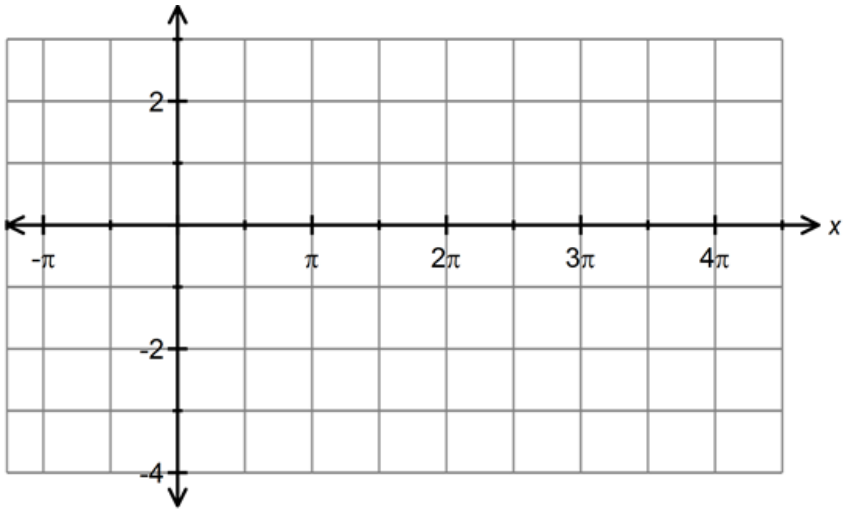
(ii) where the x-coordinate hits the sinusoidal axis going from a maximum to a minimum $\longrightarrow$ reflection on x-axis

Example 7: Sketch the graph of $y = 2\cos(x+\pi) - 1$ over two cycles.

Method 1: Use Mapping Rule

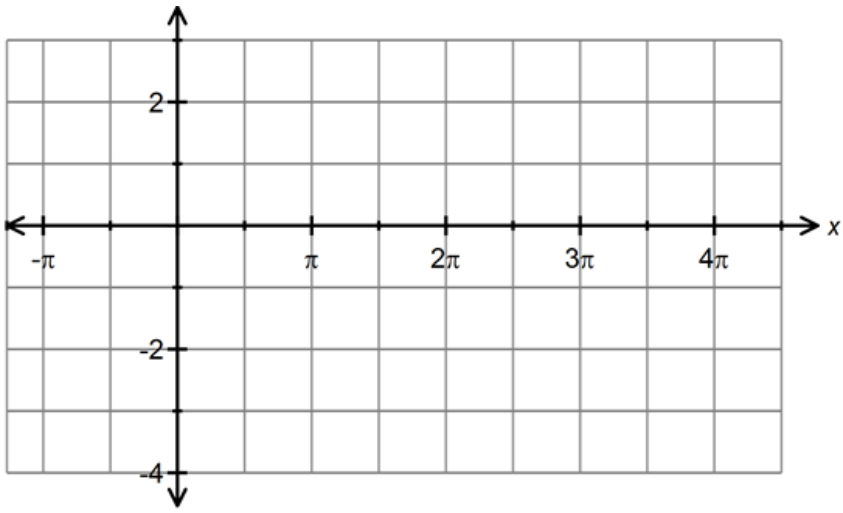
$(x, y) \rightarrow$

x	y
0	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



Example 7 cont'd $y = 2\cos(x + \pi) - 1$

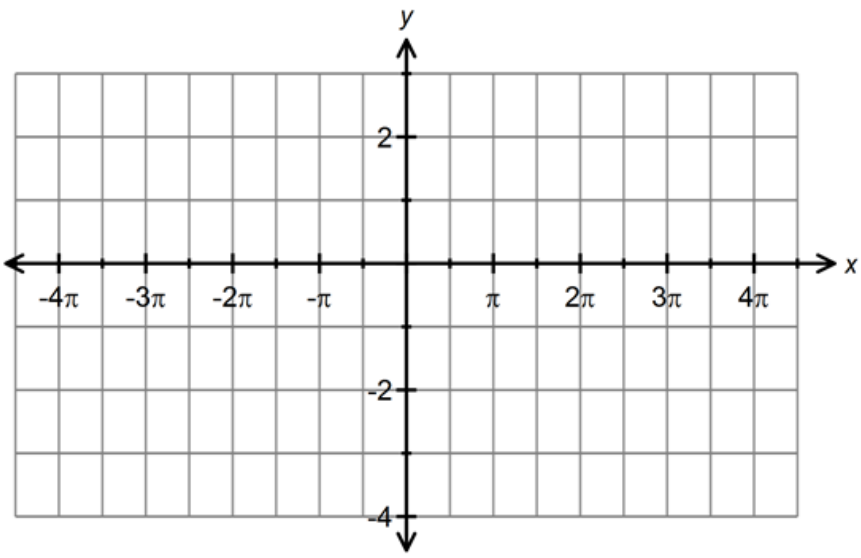
Method 2: Use Inequality



Example 8: Sketch the graph of $y = 2\cos(-(x+\pi)) - 1$ over two cycles.

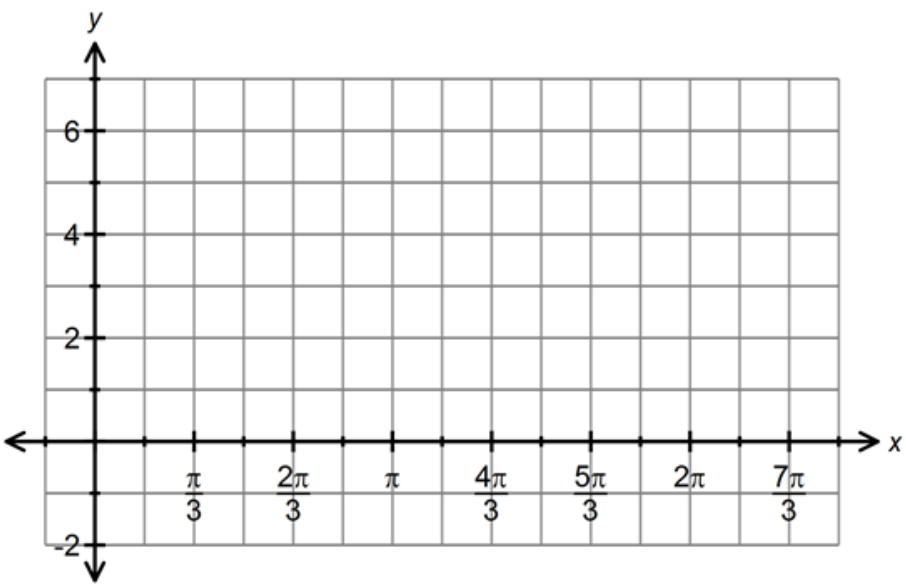
$(x, y) \rightarrow$

x	y
0	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



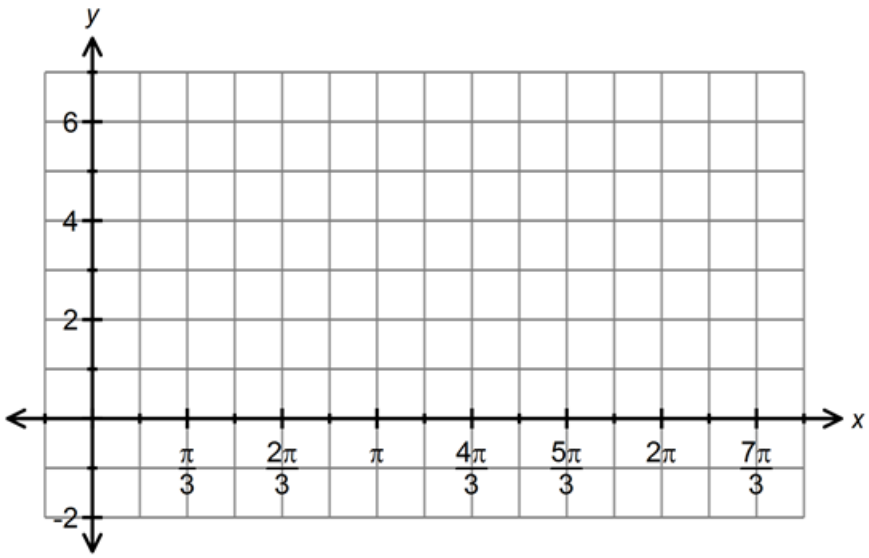
Example 9: Sketch the graph of $y = 3\sin(2x - \frac{2\pi}{3}) + 2$ over two cycles. Identify the vertical displacement, amplitude, period, phase shift, domain and range for the function.

Method 1: Use Mapping Rule

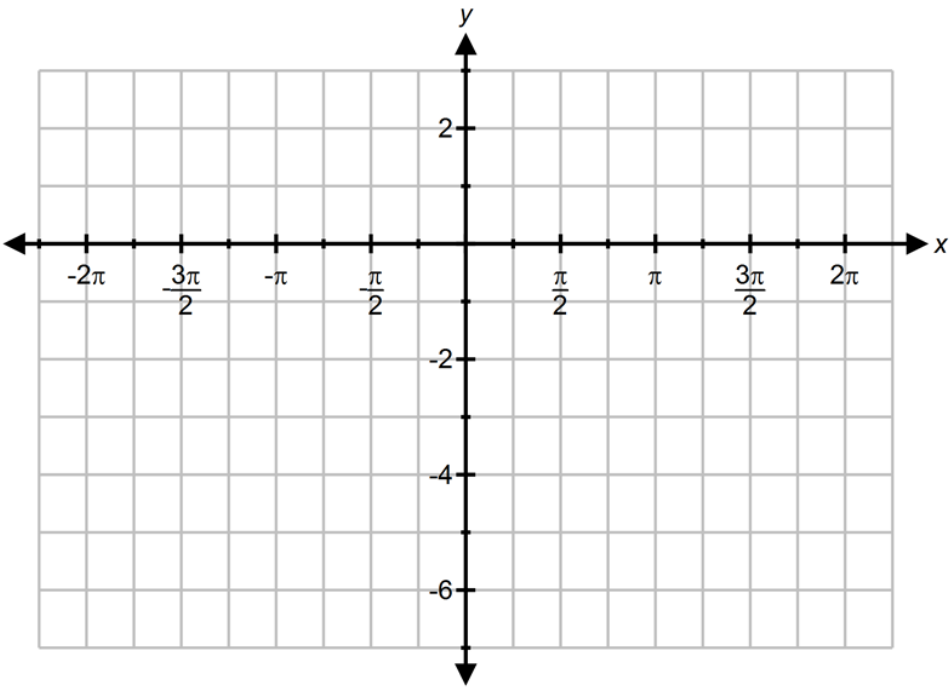


Example 9 cont'd $y = 3\sin 2(x - \frac{\pi}{3}) + 2$

Method 2: Use Inequality



Example 10: Sketch the graph of $y = -2\cos(x + \pi) - 3$ over two cycles.



Questions: p.250-252 # 1ce (no graph), 2cf (no graph), 3a, 4, 5, 6-7, 13-14

p.253 #19

Worksheet: Sketch the graphs of cosine and sine functions

Worksheet

Sketch the following functions over two cycles. Identify the vertical displacement, amplitude, period, phase shift, domain and range.

$(a)\quad y = 3\cos\left(x + \frac{\pi}{2}\right) + 2$

$(b)\quad y = 2\sin(x - \pi) + 1$

$(c)\quad y = -4\sin(2x + \pi) - 3$

$(d)\quad y = 3\cos\left(-\left(2x - \frac{\pi}{6}\right)\right) + 7$

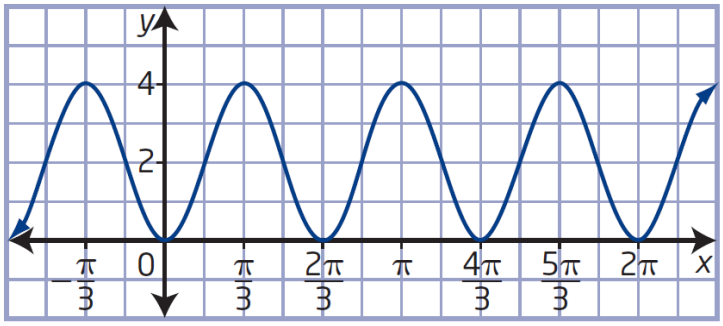
$(e)\quad y = -2\cos\left(x - \frac{\pi}{3}\right) - 4$

→

Write the Equation of the Sinusoidal Function Given the Graph.

Example 11: Write the equation of the function in the form

$$y = a\sin b(x-c)+d \quad \text{and} \quad y = a\cos b(x-c)+d$$



Identify the key characteristics of the graph and then link them to the parameters in the equation.

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maximum value =

Sinusoidal Axis =

minimum value =

amplitude =

period =

b =

Horizontal Translation:

Sine Function

phase shift ⟶ locate start of cycle

Cosine Function

phase shift ⟶ locate start of cycle

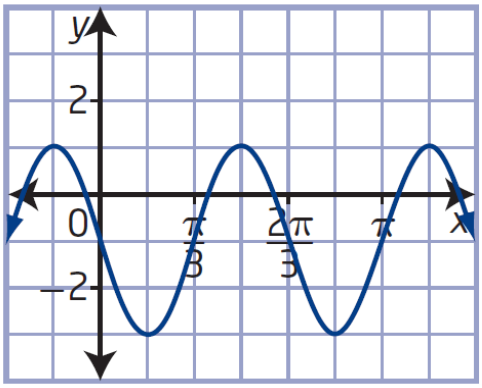
Equation:

Equation:

⟶

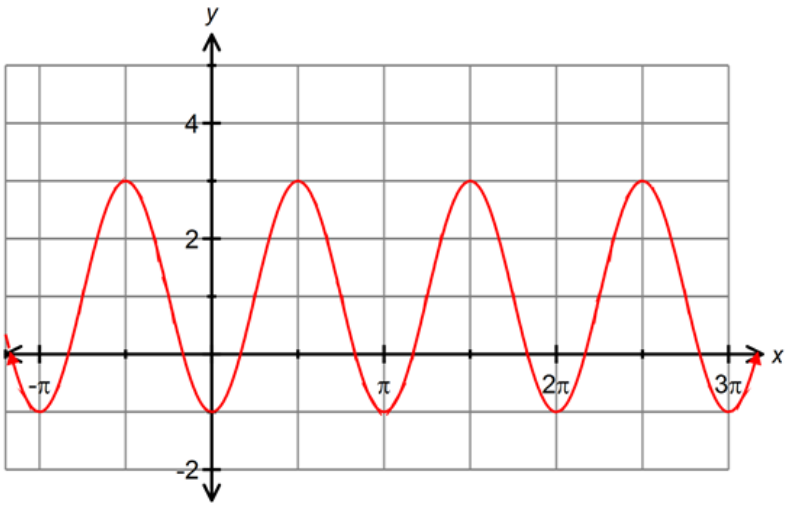
Example 12: Write the equation of the function in the form

$y = a\sin b(x-c)+d$ and $y = a\cos b(x-c)+d$



Example 13: Write the equation of the function in the form

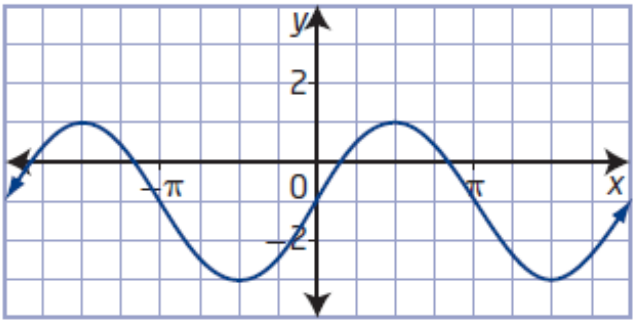
$y = a\sin b(x-c)+d$ and $y = a\cos b(x-c)+d$



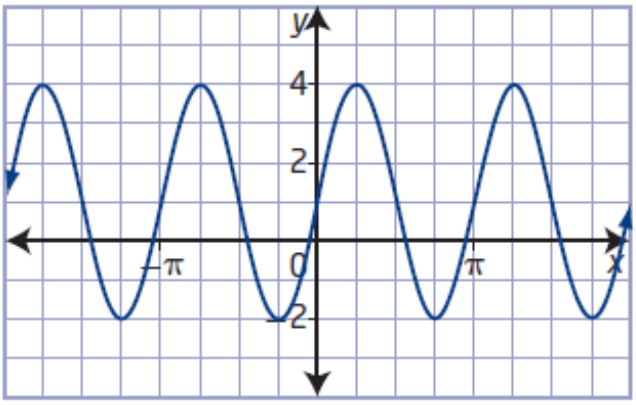
Questions: p.253 #15-16

15. Determine an equation in the form $y = a \sin b(x - c) + d$ for each graph.

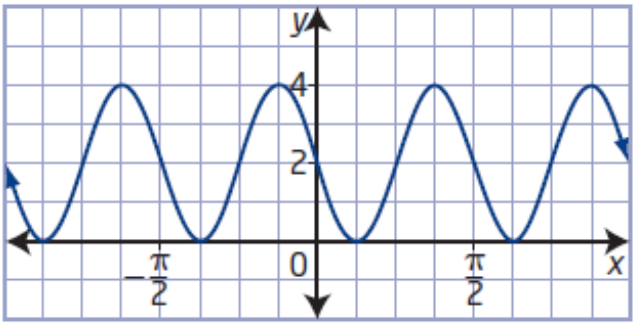
a)



b)



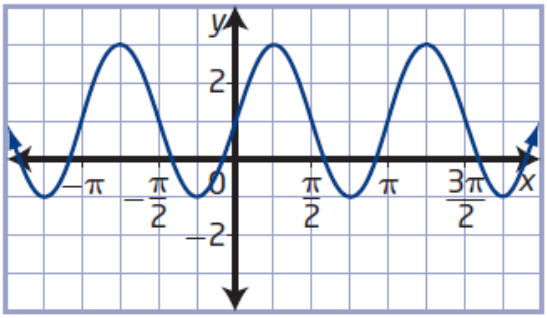
c)



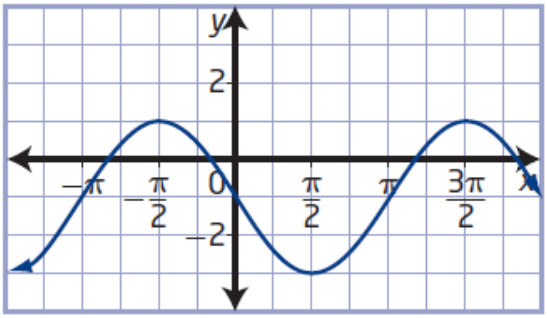
→

16. For each graph, write an equation in the form $y = a \cos b(x - c) + d$.

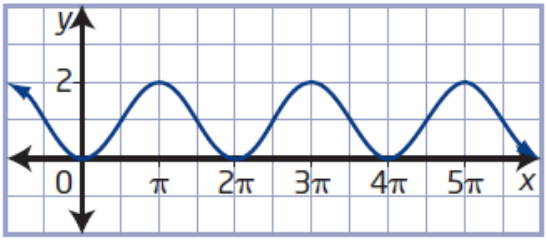
a)



b)



c)



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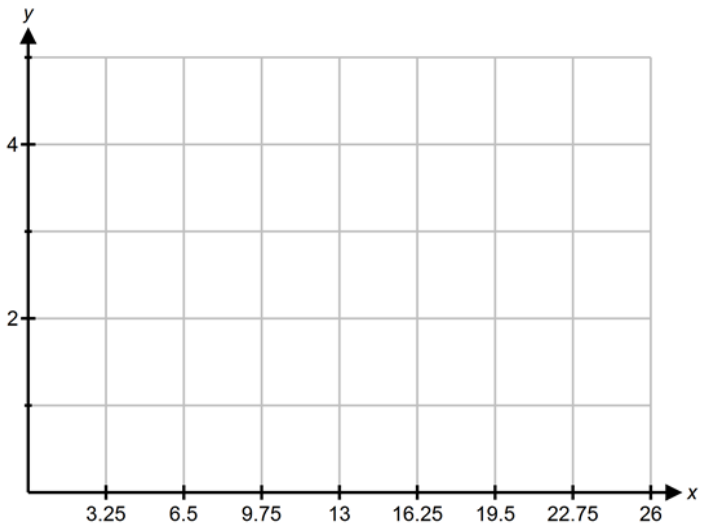
Word Problems

Example 14: The depth d, in metres, of the water in the harbor is approximated by the equation $d(t) = 0.6\cos\frac{2\pi}{13}t + 3.7$, where t is the time in hours, after the first tide.

(a) Graph the function for two cycles starting at $t = 0$.

(b) What is the period of the tide?



Inequality

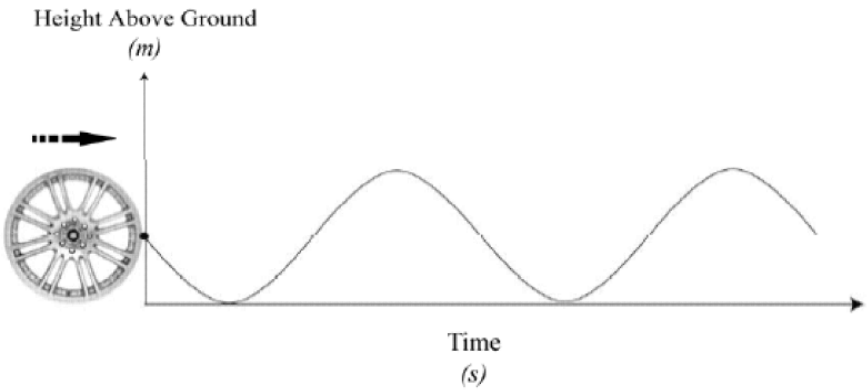
(c) If a boat requires a minimum of 3.5m of water to launch safely, for how many hours per cycle can the boat safely launch?

(d) What is the depth of the water at 7h? At what other times is the water level at this depth?

→

Example 15:

A mechanic rolls a wheel of radius 42cm and the wheel as it rolls makes 1 revolution every two seconds. The diagram below illustrates the vertical motion of a point on the wheel over time. Find an equation of the function shown as a transformation of sine or cosine.



Question: p.254 #24