Mathematics 3200
Sample Test: Function Transformations

*Section A:* Selected Response: Place the letter of your response in the space at the right. [13 points]

1. The function $y = f(x)$ is stretched horizontally by a factor of 4 and is reflected in the x-axis. What is the equation of the transformed function?

   1._____

   A) $-y = f\left(\frac{1}{4}x\right)$

   B) $y = f\left(-\frac{1}{4}x\right)$

   C) $-y = f(4x)$

   D) $y = f(-4x)$

2. The graph of $y = f(x)$ is transformed according to the mapping rule $(x, y) \to \left(2x - 3, -\frac{1}{4}y\right)$. What is the equation of the resulting function?

   2._____

   A) $y = -\frac{1}{4}f(2x - 3)$

   B) $y = -\frac{1}{4}f\left(\frac{1}{2}x + 3\right)$

   C) $y = -\frac{1}{4}f(2(x - 3))$

   D) $y = -\frac{1}{4}f\left(\frac{1}{2}(x + 3)\right)$

3. Given that $y = f(x)$ contains the point $(m, n)$, which of the following points must lie on the graph of $\frac{1}{3}y = f(x + m)$

   3._____

   A) $(2m, 3n)$

   B) $\left(2m, \frac{1}{3}n\right)$

   C) $(0, 3n)$

   D) $\left(0, \frac{1}{3}n\right)$

4. The function $y = f(x)$ is transformed to $y = 3f(4x)$. Which describes how $y = f(x)$ is transformed?

4._____

| | Horizontal Stretch | Vertical Stretch |
|---|---|---|
| A | $\dfrac{1}{4}$ | $\dfrac{1}{3}$ |
| B | $\dfrac{1}{4}$ | $3$ |
| C | $4$ | $\dfrac{1}{3}$ |
| D | $4$ | $3$ |

5. The graph of $y = f(x)$ is transformed to produce the graph of $y = -5f(2x - 6) - 1$. Which describes the horizontal translation?

5._____

A) 6 units left

B) 6 units right

C) 3 units left

D) 3 units right

6. Which of the following functions would produce a graph with the same y-intercept as the graph of $y = f(x)$?

6._____

A) $y = f(x - 4)$

B) $y = f(x) + 4$

C) $y = 3f(x)$

D) $y = f(3x)$

7. When compared to $y = f(x)$, what is the vertical stretch factor of $-2(y + 1) = f(\frac{1}{3}x)$?

7._____

A) $-2$

B) $2$

C) $-\dfrac{1}{2}$

D) $\dfrac{1}{2}$

8.   What is the inverse of $y = 2x^2 - 6$?                                                    8._____

A)   $x = \pm\sqrt{\dfrac{y+6}{2}}$

B)   $x = \pm\sqrt{\dfrac{1}{2}y + 6}$

C)   $y = \pm\sqrt{\dfrac{x+6}{2}}$

D)   $y = \pm\sqrt{\dfrac{1}{2}x + 6}$

9.   The function $y = f(x)$ is stretched vertically by a factor of 3, and translated 5 units      9._____
     right and 1 unit up.  What is the equation of the resulting function?

A)   $y = \dfrac{1}{3}f(x+5) - 1$                        B)   $y = \dfrac{1}{3}f(x-5) + 1$

C)   $y = 3f(x+5) - 1$                                   D)   $y = 3f(x-5) + 1$

10.  Which combination of transformations is required to map $y = f(x)$ onto                        10._____
     $y = \dfrac{1}{2}f(-x)$?

     A)  Reflection in the x-axis, Stretched vertically by a factor of 2
     B)  Reflection in the x-axis, Stretched vertically by a factor of $\dfrac{1}{2}$
     C)  Reflection in the y-axis, Stretched vertically by a factor of 2
     D)  Reflection in the y-axis, Stretched vertically by a factor of $\dfrac{1}{2}$

11.  The domain of $y = f(x)$ is $\{x / {-12} \le x \le 6, x \in \mathbb{R}\}$.  What is the domain of      11._____
     $h(x) = 3f(2x)$

A)   $\{x / {-24} \le x \le 12, x \in \mathbb{R}\}$          B)   $\{x / {-6} \le x \le 3, x \in \mathbb{R}\}$
C)   $\{x / {-4} \le x \le 2, x \in \mathbb{R}\}$           D)   $\{x / {-36} \le x \le 18, x \in \mathbb{R}\}$

12. The function $y = f(x)$ has zeroes $x = -4$ and $x = 2$. What are the zeroes of the     12._____
function $g(x) = 3f\left(-\frac{1}{2}(x-2)\right)$?

    A)   $x = 2$ and $x = -1$                           B)   $x = 10$ and $x = -2$
    C)   $x = 0$ and $x = -3$                           D)   $x = -12$ and $x = 6$

13. If $f^{-1}(x)$ contains the point (-3, 5), which point must lie on $f(x)$?          13. _____

    A) (3, -5)                 B) $\left(-\frac{1}{3}, \frac{1}{5}\right)$

    C) (5, -3)                 D) (-5, 3)

***Section B:*** Constructed Response. Be sure to show all workings in order to receive full marks.

14.     The point (-4, 10) lies on the graph of $f(x)$. What is the image point on the graph
       of   $y = -3f(2x-6) + 1$?                                      [2 points]

15.     The function $y = f(x)$ is transformed to produce a function of the form $y = af\left(b(x-h)\right) + k$.
       The list of transformations is given below. Write the mapping rule that represents this set of
       transformations and then write the function in the form $y = af\left(b(x-h)\right) + k$.
                                                            [4 points]

- Horizontal Stretch by a factor of 3
- Vertical stretch by a factor of 2
- Reflection in the y-axis
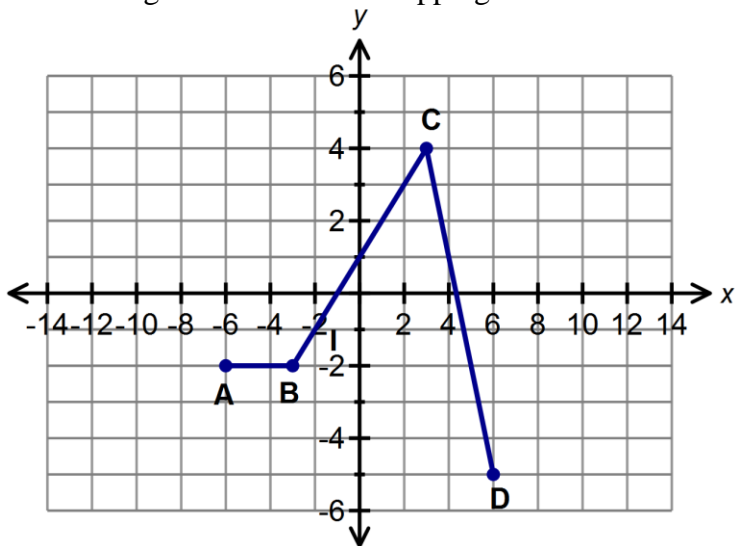- Translation of 7 units right and 1 unit up

16. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.
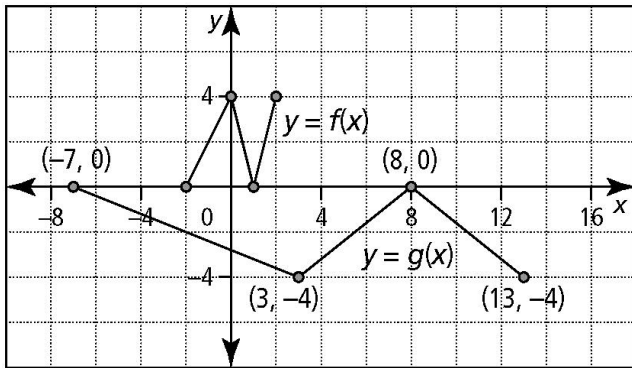
[4 points]



17. The graph of $y = f(x)$ is shown. Sketch the graph of $-2y + 4 = f\left(\frac{1}{2}(x - 1)\right)$ on the same grid and write the mapping rule that describes this set of transformations.

[4 points]

18. Determine algebraically, the inverse of the function $f(x) = (x-2)^2 + 3$. Restrict the domain so that the inverse of $f(x)$ is also a function. Verify by sketching the graph of the restricted function and its inverse.

[4 points]