Mathematics 3200 Unit 5: Trigonometric Functions and Graphs

Name: Key /25

Part A: Selected Response: Place the letter of the correct response in the space provided. (12 marks)

1. What is the period of $y = 3\cos\frac{1}{2}(x-\pi)$? 1. C

$b = \frac{1}{2} \therefore HS = 2$

period = H.S. × 2π
= 2 × 2π
= 4π

(A) $\frac{\pi}{2}$ (B) π

(C) 4π (D) 8π

2. What is the range of the function $y = \frac{1}{2}\cos 2\left(x-\frac{\pi}{4}\right)-3$? 2. B

$y = -3 + \frac{1}{2} = -\frac{5}{2}$

$y = -3 - \frac{1}{2} = -\frac{7}{2}$

(A) $\{y \mid -5 \le y \le -1,\ y \in R\}$

(B) $\{y \mid -\frac{7}{2} \le y \le -\frac{5}{2},\ y \in R\}$

(C) $\{y \mid 1 \le y \le 5,\ y \in R\}$

(D) $\{y \mid \frac{5}{2} \le y \le \frac{7}{2},\ y \in R\}$

3. What are the transformations of the graph $y = \sin(3\theta - \pi)$? 3. A

$3(\theta - \frac{\pi}{3})$

(A) horizontal stretch by a factor of $\frac{1}{3}$ and a horizontal shift of $\frac{\pi}{3}$ units right

(B) horizontal stretch by a factor of $\frac{1}{3}$ and a horizontal shift of π units right

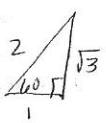
(C) horizontal stretch by a factor of 3 and a horizontal shift of $\frac{\pi}{3}$ units right

(D) horizontal stretch by a factor of 3 and a horizontal shift of π units right

4. Solve for x: $2\sin\theta - \sqrt{3} = 0$ where $0 \le x \le 2\pi$. 4. D

$2\sin\theta = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$



$\theta = 60°$

sin pos in Q1 + Q2

$\therefore \theta = 60 + \theta = 180 - 60$

$= 120°$

$\frac{\pi}{3}$ $\frac{2\pi}{3}$

(A) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ (B) $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

(C) $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$ (D) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

5. If the point $\left(\frac{\pi}{2}, -2\right)$ lies on the graph of $y = a\cos\left(x - \frac{\pi}{4}\right) - 4$, what is the value of a?

5. A

(A) $2\sqrt{2}$ (B) $\frac{\sqrt{2}}{2}$

(C) 2 (D) $\sqrt{2}$

6. The range of a trigonometric function of the form $y = a\sin b(x-c)+d$ is $\{y \mid -2 \le y \le 8,\ y \in R\}$. What is the value of d?

6. B

(A) 1 (B) 3

(C) 5 (D) 8

7. What is the maximum value of $y = 2\cos 4\left(x - \frac{\pi}{6}\right) - 5$?

7. B

(A) -7 (B) -3

(C) 3 (D) 7

8. The partial graph of a trigonometric function is shown. The graph has a maximum value at $\left(\frac{\pi}{2}, 112\right)$ and a minimum value at $\left(\frac{3\pi}{2}, 28\right)$.

Which equation can be used to represent this graph?

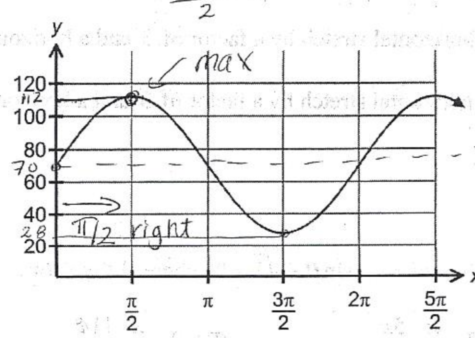
8. C

(A) $y = 70\cos\left(x - \frac{\pi}{2}\right) + 42$

(B) $y = 42\cos(x - 2\pi) + 70$

(C) $y = 42\cos\left(x - \frac{\pi}{2}\right) + 70$

(D) $y = 70\cos(x - 2\pi) + 42$



9. Write the equation of the sine function if the amplitude is 3 and the period is $\frac{5\pi}{6}$?

9. D

(A) $y = 3\sin\frac{5\pi}{6}x$ (B) $y = 3\sin\frac{6}{5\pi}x$

(C) $y = 3\sin\frac{5}{12}x$ (D) $y = 3\sin\frac{12}{5}x$

10. What is the domain of $y = \tan\theta$? 10. C

(A) $x \mid x \neq \frac{\pi}{4} + \pi k, k \in I$

(B) $x \mid x \neq \frac{\pi}{4} + 2\pi k, k \in I$

(C) $x \mid x \neq \frac{\pi}{2} + \pi k, k \in I$

(D) $x \mid x \neq \frac{\pi}{2} + 2\pi k, k \in I$

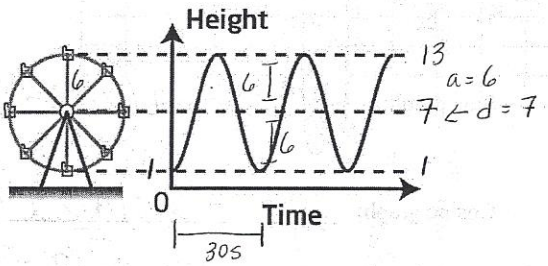
11. A Ferris wheel with a radius of 6m rotates once every 30 seconds. Passengers get on board at a point 1m above the ground at the bottom of the Ferris wheel. Which function models this situation? 11. A

(A) $y = -6\cos\frac{\pi}{15}x + 7$

(B) $y = -6\cos\frac{15}{\pi}x + 7$

(C) $y = -\frac{1}{6}\cos\frac{\pi}{15}x + 7$

(D) $y = -\frac{1}{6}\cos\frac{15}{\pi}x + 7$



a = 6
d = 7

period = HS x 2π

$\frac{30}{2\pi}$ = HS = $\frac{15}{\pi}$

∴ b = π/15

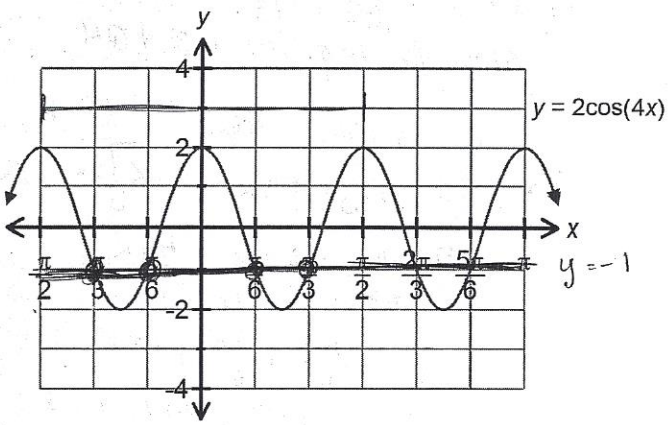
12. Given the graph below, what is the solution for $2\cos 4x = -1$ where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$? 12. C

(A) $x = \pm\frac{\pi}{3}, \pm\frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}$

(B) $x = \pm\frac{\pi}{2}, 0$

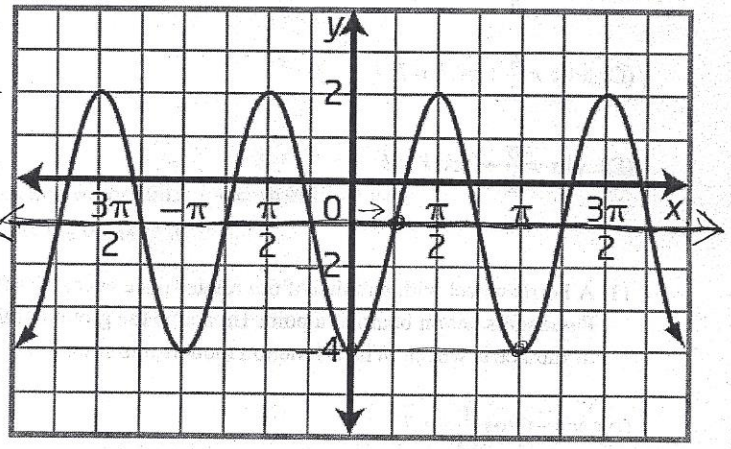
(C) $x = \pm\frac{\pi}{3}, \pm\frac{\pi}{6}$

(D) $x = \pm\frac{\pi}{2}, 0, \pi$



Part B: Constructed Response: Show workings to all problems.

/4 13. Write the equation for the graph shown in the form $y = a\sin b(x-c)+d$ and in the form $y = a\cos b(x-c)+d$.



Period = π

Period = HS · 2π

HS = $\frac{\text{Period}}{2\pi}$

= $\frac{\pi}{2\pi} = \frac{1}{2}$ ∴ $b = 2$

Sine graph: $y = 3\sin\left[2(x-\pi/4)\right]-1$

Cosine graph: $y = -3\cos 2x - 1$

OR $y = 3\cos\left[2(x-\pi/2)\right]-1$

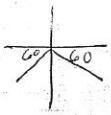
/4 14. Determine all solutions, in radian measure, for the equation $\sin\left[\frac{1}{2}\left(\theta-\frac{\pi}{2}\right)\right] = -\frac{\sqrt{3}}{2}$

Let $w = \frac{1}{2}[\theta - \pi/2]$

$\sin w = -\frac{\sqrt{3}}{2}$

ref ∠: $w = 60° = \pi/3$

sin is neg. in Q3 + Q4



∴

$w = \pi + \pi/3 = 4\pi/3$ + $w = 2\pi - \pi/3 = \frac{5\pi}{3}$

$2 \times \frac{1}{2}[\theta - \pi/2] = [4\pi/3 + 2\pi K,] \times 2$ $2 \times \frac{1}{2}[\theta - \pi/2] = [5\pi/3 + 2\pi K] \times 2$

$\theta - \pi/2 = 8\pi/3 + 4\pi K$ $\theta - \pi/2 = \frac{10\pi}{3} + 4\pi K$

$+\pi/2$ $+\pi/2$ $+\pi/2$ $+\pi/2$

$\theta = \frac{16\pi}{6} + \frac{3\pi}{6} + 4\pi K$ $\theta = \frac{20\pi}{6} + \frac{3\pi}{6} + 4\pi K$

$\theta = \frac{19\pi}{6} + 4\pi K, K \in I$ $\theta = \frac{23\pi}{6} + 4\pi K, K \in I$

Solutions:

$$\theta = \begin{cases} \frac{19\pi}{6} + 4\pi K, K \in I \\ \frac{23\pi}{6} + 4\pi K, K \in I \end{cases}$$

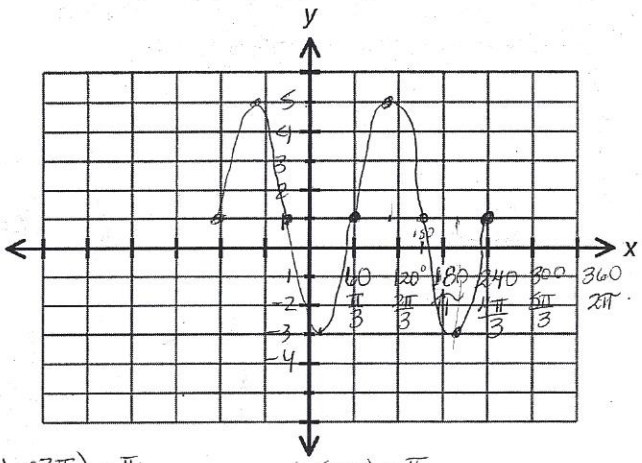
/5 15. Sketch the graph of the function $y = 4\sin 2\left(x - \frac{\pi}{3}\right) + 1$.

State the domain and the range.

mapping rule: $(x, y) \to \left(\frac{1}{2}x + \frac{\pi}{3}, 4y + 1\right)$

	x	y
0	0	0
$\pi/2$	90°	1
π	180°	0
$3\pi/2$	270°	-1
2π	360°	0

x	y
60° $\pi/3$	1
105° $7\pi/12$	5
150° $5\pi/6$	1
195° $13\pi/12$	-3
240° $4\pi/3$	1



$\frac{1}{2}\left(\frac{\pi}{2}\right) + \pi/3$

$\pi/4 + \pi/3$

$\frac{3\pi}{12} + \frac{4\pi}{12}$

$= \frac{7\pi}{12}$

$\frac{1}{2}(\pi) + \pi/3$

$\pi/2 + \pi/3$

$\frac{3\pi}{6} + \frac{2\pi}{6}$

$= \frac{5\pi}{6}$

$\frac{1}{2}\left(\frac{3\pi}{2}\right) + \pi/3$

$\frac{3\pi}{4} + \pi/3$

$\frac{9\pi}{12} + \frac{4\pi}{12}$

$13\pi/12$

$\frac{1}{2}(2\pi) + \frac{\pi}{3}$

$\frac{3\pi}{3} + \frac{\pi}{3}$

$= \frac{4\pi}{3}$

Domain: $x \in R$ Range: $-3 \leq y \leq 5, y \in R$