

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Part A: Selected Response:** Place the letter of the correct response in the space provided.

(13 marks)

$$2^3 + 7(2)^2 - 8 + 3 - K = 0$$

$$8 + 28 - 8 + 3 - K = 0$$

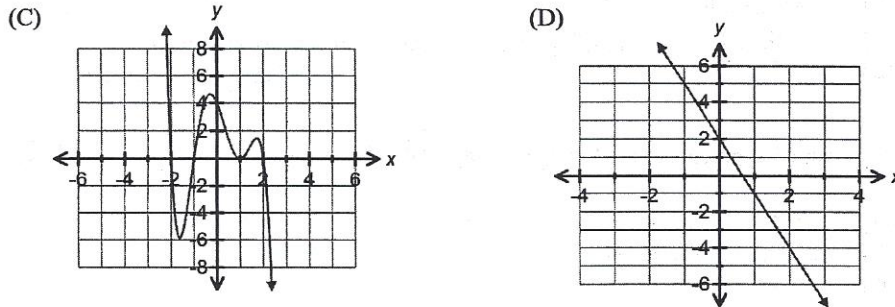
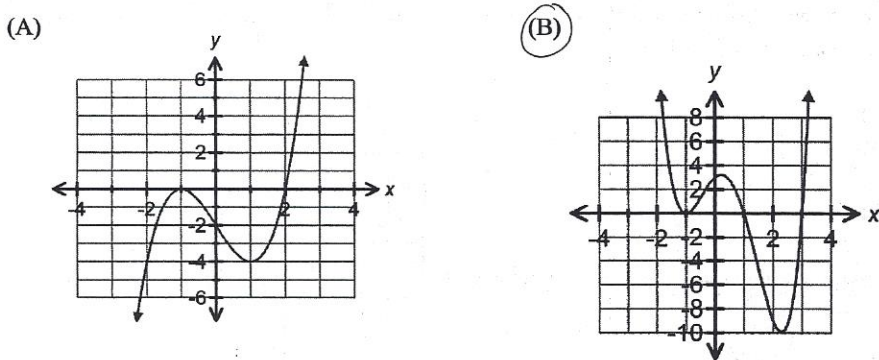
$$31 - K = 0$$

$$K = 31$$

1. If  $(x-2)$  is a factor of  $x^3 + 7x^2 - 4x + (3-k)$ , what is the value of  $k$ ? 1. D

- (A) -31    (B) -28    (C) 28    (D) 31

2. Which graph below represents the graph of an even degree function? 2. B

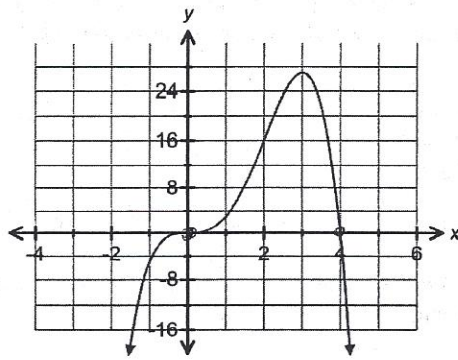


3. Which of the equations below is best represented by the given graph? 3. A

- (A)  $y = -x^4 + 4x^3$   
 (B)  $y = x^4 - 4x^3$   
 (C)  $y = x^3 - 4x^2$   
 (D)  $y = -x^3 + 4x^2$

$$-x^3(x-4)$$

$$-x^4 + 4x^3$$



4. Which statement is true for a polynomial function? 4. D

- (A) All even degree polynomial functions have at least one  $x$ -intercept.  
 (B) Some odd degree polynomial functions have no  $x$ -intercepts.  
 (C) Even degree polynomial functions always have an even number of  $x$ -intercepts.  
 (D) All odd degree polynomials have at least one  $x$ -intercept.

5. Which function has each of the characteristics:

5. C

- ✓ an even function
- ✓ end behavior in the third and fourth quadrants
- ✓ y-intercept is -6  $\therefore a$  is neg.

- (A)  $P(x) = x^4 - 5x^2 - 6$
- (B)  $P(x) = -x^4 + 3x^3 + 6$
- (C)  $P(x) = -(x+2)(x+3) = -(x^2 + 5x + 6)$
- (D)  $P(x) = -x^3 + x - 6 = -x^2 - 5x - 6$

6. Which represents the value of  $k$  if the remainder is 5 for  $(2x^3 + 4x^2 + kx - 3) \div (x + 1)$ ?

6. A

- (A) -6
- (B) -2
- (C) 2
- (D) 6

$$\begin{aligned} P(-1) &= 5 \\ -2 + 4 - k - 3 &= 5 \\ -k - 1 &= 5 \\ -k &= 6 \\ k &= -6 \end{aligned}$$

7. What are the  $x$ -intercepts of  $y = 4x^3 - 12x^2 + 8x$ ?

7. C

- (A)  $x = -4, -2, -1$
- (B)  $x = -2, -1, 0$
- (C)  $x = 0, 1, 2$
- (D)  $x = 1, 2, 4$

$$\begin{aligned} 4x(x^2 - 3x + 2) \\ 4x(x-2)(x-1) \\ x = 0, 1, 2 \end{aligned}$$

8. List all possible integral zeros for  $P(x) = x^4 + 3x^3 - 2x^2 - 12x - 8$ .

8. C

- (A)  $\pm 1, \pm 8$
- (B)  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
- (C)  $\pm 1, \pm 2, \pm 4, \pm 8$
- (D)  $\pm 2, \pm 4$

9. The volume of a rectangular prism is  $V = 2x^3 - 5x^2 - x + 6$ . If two of the dimensions are  $x - 2$  and  $x + 1$ , what is an expression for the other dimension?

9. C

- (A)  $x - 6$
- (B)  $x - 6$
- (C)  $2x - 3$
- (D)  $2x + 3$

$$\begin{array}{r} 2 \overline{) 2 \quad -5 \quad -1 \quad 6} \\ \underline{4 \quad -2 \quad -6} \\ -1 \quad -3 \quad 10 \\ -1 \quad -3 \quad 10 \\ \underline{-2 \quad 3} \\ 2 \quad -3 \quad 10 \end{array}$$

10. What are the  $x$ -intercepts of  $y = x^2(x+3)(x-2)$ ?

10. C

- (A) -3 and 2
- (B) 3 and -2
- (C) 0, -3, and 2
- (D) 0, 3, and -2

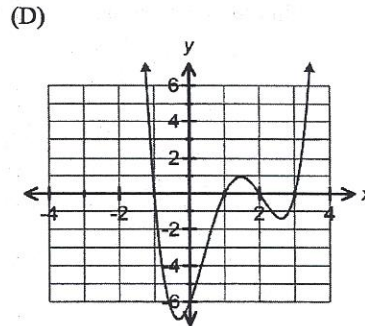
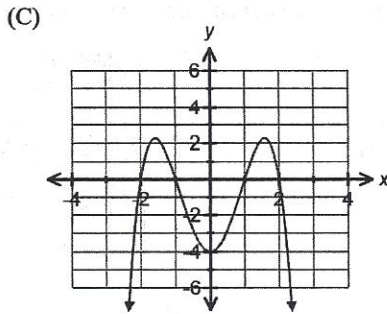
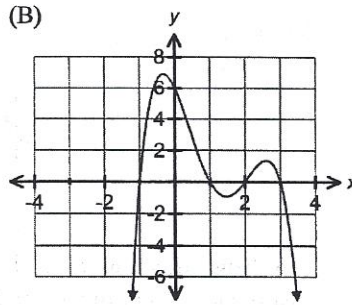
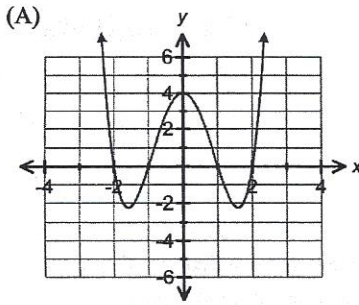
$$0, -3, 2$$

11. What is the quotient and remainder for  $(2x^3 - x^2 + 2x + 4) \div (x - 3)$ ?

11. C

12. Which sketch best represents the graph of  $y = ax^4 + bx^3 + cx^2 + dx + e$  if  $a > 0$  and  $e < 0$ ?

12. D



13. How many  $x$ -intercepts are possible for the polynomial function  $P(x) = ax^5 + bx^4 + cx^3$ ?

13. D

- (A) 1
- (B) 3
- (C) 4
- (D) 5

**Part B: Constructed Response:** Show workings to all problems.

14. For what value of  $k$  will the polynomial  $P(x) = 4x^3 - 3x^2 + kx + 6$  have the same remainder when it is divided by both  $x - 1$  and  $x + 3$ ?

/3

$$\begin{aligned}
 P(1) &= P(-3) \\
 4(1)^3 - 3(1)^2 + k(1) + 6 &= 4(-3)^3 - 3(-3)^2 + k(-3) + 6 \\
 k + 7 &= -108 - 27 - 3k + 6 \\
 4k &= -136 \\
 k &= -34
 \end{aligned}$$

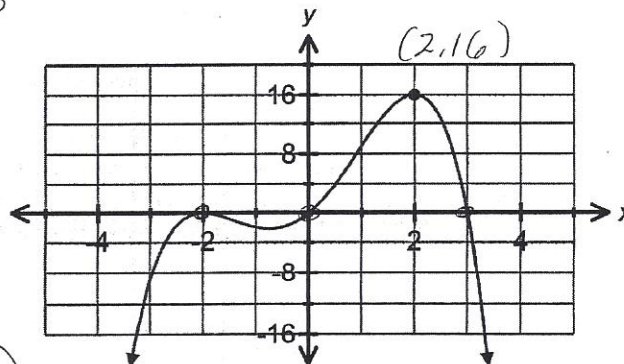
15. Given the graph, determine the equation of the polynomial in factored form.

/3

$x$ -ints:  $x = -2$  ( $m=2$ ),  $0$ ,  $3$

$$\begin{aligned}
 y &= a(x+2)^2(x)(x-3) \\
 16 &= a(2+2)^2(2)(2-3) \\
 16 &= a(4)(2)(-1) \\
 16 &= -8a \\
 -2 &= a
 \end{aligned}$$

3  $y = -2x(x+2)^2(x-3)$



16. Give that  $x=2$  is a root of the function,  $P(x) = 2x^4 - 3x^3 - 6x^2 + 5x + 6$ , determine the other roots. /4

$$\begin{array}{r} 2 \overline{) 2 \quad -3 \quad -6 \quad 5 \quad 6} \\ \underline{4 \quad 2 \quad -8 \quad -6} \\ -1 \overline{) 2 \quad 1 \quad -4 \quad -3 \quad 10} \\ \underline{-2 \quad 1 \quad 3} \\ 2 \quad -1 \quad -3 \quad 10 \end{array}$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$x = \frac{3}{2} \quad x = -1, x = -1$$

$(m=2)$

17. The height of a square-based box is 4 cm more than the side length of its square base. The volume of the box is  $225 \text{ cm}^3$ . Create an equation to represent this situation and use it to algebraically determine the dimensions of the box? /4

$$V = 225$$

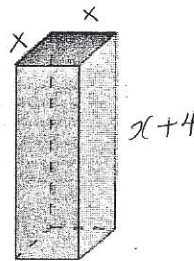
$$l \cdot w \cdot h = 225$$

$$x \cdot x(x+4) = 225$$

$$x^3 + 4x^2 - 225 = 0$$

$x = 5$

Dimensions:  
 $5 \times 5 \times 9$



$$\begin{array}{r} 5 \overline{) 1 \quad 4 \quad 0 \quad -225} \\ \underline{5 \quad 45 \quad 225} \\ 1 \quad 9 \quad 45 \quad 10 \end{array}$$

$$x^2 + 9x + 45$$

$$x = \frac{-9 \pm \sqrt{81 - 4(45)}}{2}$$

$$= \frac{-9 \pm \sqrt{-99}}{2}$$

imaginary roots  
extraneous roots.

18. For the polynomial function  $f(x) = -\frac{1}{2}(x+3)(x+1)^2(x-2)^2$  determine the following characteristics: /6

(i) the zeros:  $x = -3, -1(m=2), 2(m=2)$

(ii) the y-intercept:  $y = -6$

$$f(0) = -\frac{1}{2}(0+3)(0+1)^2(0-2)^2$$

$$= -\frac{1}{2}(3)(1)(4)$$

$$= -6$$

(iii) degree of the function:  $5$

(iv) sketch the graph

(v) the intervals where the function is positive and negative:

Positive:  $x < -3$  or  $(-1, 2)$

Negative:  $x > -3, x \neq -1, 2$  or  $(-3, -1), (1, 2), (2, \infty)$

