Mathematics 3200

January 2104 Midterm Exam

Answer Key

Part 1: Multiple Choice

1. C
2. B
3. A
4. C
5. C
6. A
7. A
8. B
9. B
10. D
11. D
12. A
13. B
14. B
15. B
16. D
17. A
18. C
19. C
20. C
21. D
22. D
23. D
24. B
25. C
26. D

Part 2: Constructed Response

1. Mapping Rule: $(x, y) \rightarrow (2x+1, -y-3)$ (1 point)

Apply mapping rule to several points: (0.5 points each = 2 points)

$(-1,3) \rightarrow (-1,-6)$ $(0,4) \rightarrow (1,-7)$ $(1,1) \rightarrow (3,-4)$

$(2,1) \rightarrow (5,-4)$

Sketch new points on second grid. (1 point)

2. $y = f\left(\frac{1}{4}(x-3)\right)$ has a HS of 4 and a HT of 3 to the right

$y = f\left(\frac{1}{4}x-3\right) = f\left(\frac{1}{4}(x-12)\right)$ has a HS of 4 and a HT of 12 to the right

Possible answer: Both functions have the same horizontal stretch factor (HS) but different horizontal translations (HT).

3. $y = \frac{1}{3}(x-5)^2+1$ (Change f(x) to y) Note: V(5, 1)

$x = \frac{1}{3}(y-5)^2+1$ (Interchange x and y)

$y = 5 \pm \sqrt{3(x-1)}$ (Solve for y)

If $x \geq \overset{1}{\cancel{5}}$, then $f^{-1}(x) = 5+\sqrt{3(x-1)}$

If $x \leq \overset{1}{\cancel{5}}$, then $f^{-1}(x) = 5-\sqrt{3(x-1)}$

Suggest:

2 points for finding inverse,

1 point for each correct final statements with restricted domains on f(x).

4. Students need to determine where the radicand is positive:

$-2x^2 + 4x + 16 \geq 0 \ \rightarrow -2(x^2 - 2x - 8) = 0 \ \rightarrow -2(x-4)(x+2) = 0$

Roots are $x = -2$ and $x = 4$ (2 points)

Using a "sign chart" or graphing knowledge, solution is $\{x|-2 \leq x \leq 4\}$

(0.5 points)

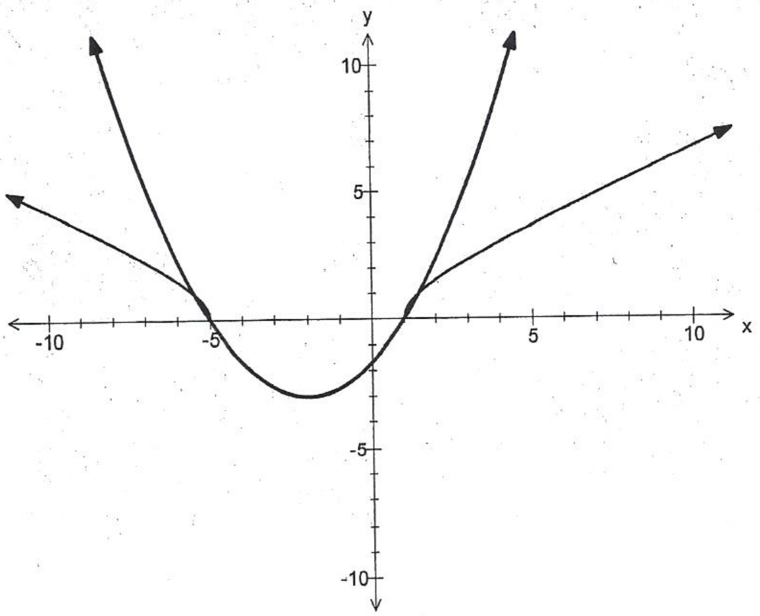
Students need to find the vertex ($x = -\frac{b}{2a}$) : V(1, 18)

(1 point)

Therefore, the range will be $\{y|0 \leq y \leq \sqrt{18}\}$ or $\{y|0 \leq y \leq 3\sqrt{2}\}$

(0.5 points)

5. A.



B. Domain: $\{x|x \geq 1 \ and \ x \leq -5, x \in \mathcal{R}\}$

Range: $\{y|y \geq 0\}$

C. $y = \sqrt{f(x)}$ is undefined when $f(x) < 0$. This happens when $-5 < x < 1$.

6. List possible roots: $\pm\{1,2,3,4,6,8,12,24\}$

Using the remainder theorem and/or synthetic division, $x = -1$, $x = -4$, and $x = 6$ are the roots.

7. $P(x) = a(x+2)^2(x-1)(x-3)$ (2 points)

Solve for "a" by substituting the coordinates of a point, say (0,2):

$2 = a\ (0+2)^2(0-1)(0-3)$

$2 = 12a$

$a = \frac{1}{6}$ (1.5 points)

Therefore, $P(x) = \frac{1}{6}(x+2)^2(x-1)(x-3)$ (0.5 points)

8. Draw a reference triangle in quadrant 2.

Hypotenuse, either through use of triples or Pythagorean Theorem, is 13

(1 point)

$\sin\theta = \frac{y}{r} = \frac{12}{13}$ (1 point)

$\sec\theta = \frac{r}{x} = -\frac{13}{5}$ (1 point)

9. Let $x = \csc\theta$

$$4x^2 = -4x + 15$$

$$4x^2 + 4x - 15 = 0$$

$$(2x - 3)(2x + 5) = 0$$

$x = \frac{3}{2} \quad x = -\frac{5}{2}$ (1 point)

$\csc\theta = \frac{3}{2} \rightarrow \sin\theta = \frac{2}{3} \rightarrow ref\ angle = 0.73$ (0.5 point)

$\Rightarrow$ $\theta = 0.73, 2.41, \cancel{-}3.87, \cancel{-}5.55$ (1 point)

$\csc\theta = -\frac{5}{2} \rightarrow \sin\theta = -\frac{2}{5} \rightarrow ref\ angle = 0.41$ (0.5 point)

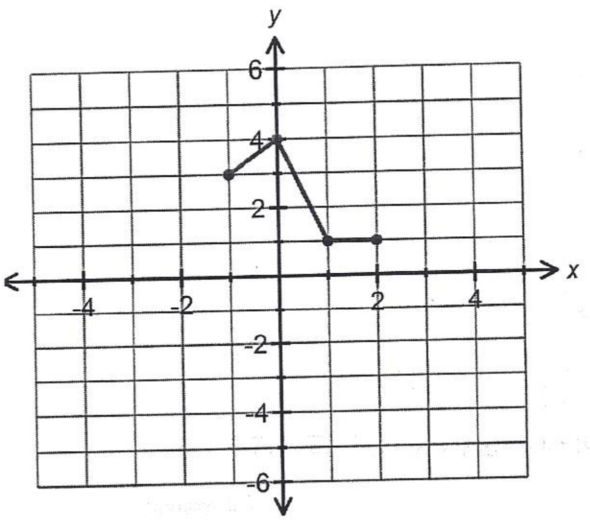
$\theta = -0.41, -2.73, 3.55, 5.87$ (1 point)

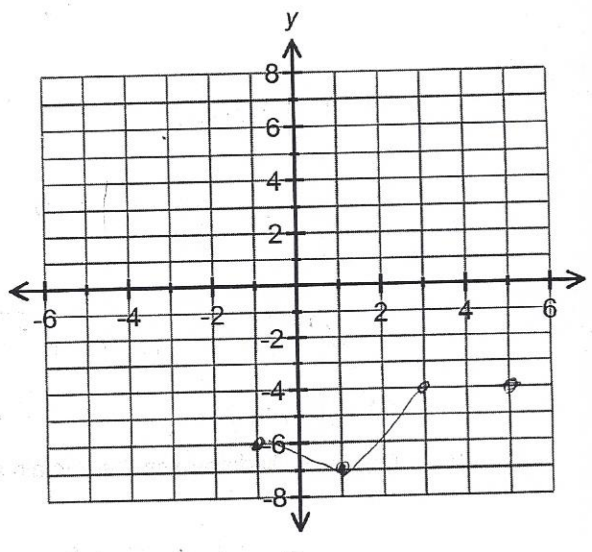
Part 2: Constructed Response:

Answer all questions in the space provided and show all necessary workings. (34 marks)

27. Given the graph of $f(x)$, sketch and label the graph of the transformed graph $y+3=-f\left(\frac{1}{2}(x-1)\right)$ and write the mapping rule.

(4 marks)





$(x,y) \rightarrow (2x+1, -y-3)$

x	y
-1	3
0	4
1	1
2	1

$\rightarrow$

$2x+1$	$y-3$
-1	-6
1	-7
3	-4
5	-4

28. Explain how the transformation described by $y=f\left(\frac{1}{4}x-3\right)$ and $y=f\left(\frac{1}{4}(x-3)\right)$ are similar and how they are different.

(2 marks)

$f\left(\frac{1}{4}x-3\right)$ vs. $f\left(\frac{1}{4}(x-3)\right)$

$= f\left(\frac{1}{4}(x-12)\right)$

Both have a horizontal stretch factor of 4 but different horizontal translations, one shifted 12 units right, the other shifted 3 units right.

29. Algebraically determine $f^{-1}(x)$ if $f(x) = \frac{1}{3}(x-5)^2 + 1$. Determine a restriction on the domain of the function in order for its inverse to be a function.

(3 marks)

$y = \frac{1}{3}(x-5)^2 + 1$

$x = \frac{1}{3}(y-5)^2 + 1$

$x - 1 = \frac{1}{3}(y-5)^2$

$3(x-1) = (y-5)^2$

$\pm\sqrt{3(x-1)} = y - 5$

$y = 5 \pm \sqrt{3(x-1)}$

$f^{-1}(x) = 5 + \sqrt{3(x-1)}\ ,\ x \geq 1$

OR $f^{-1}(x) = 5 - \sqrt{3(x-1)}\ ,\ x \leq 1$

30. Algebraically determine the domain and range of $y = \sqrt{-2x^2 + 4x + 16}$.

(4 marks)

$-2x^2 + 4x + 16 \geq 0$

$-2(x^2 - 2x - 8) \geq 0$

$-2(x-4)(x+2) \geq 0$

$x = 4 \quad x = -2$

$\overset{-}{} \quad \overset{+}{} \quad \overset{-}{}$

$-2 \qquad 4$

Domain: $-2 \leq x \leq 4$

Range: $0 \leq y \leq \sqrt{18}$

vertex:

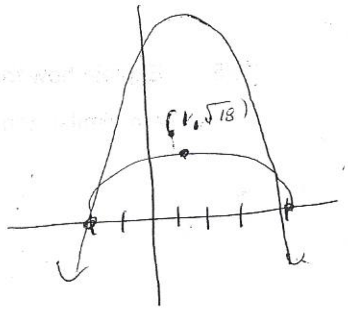
$x = \frac{-b}{2a} = \frac{-4}{-(-2)} = 1$

$y = -2(1)^2 + 4(1) + 16$

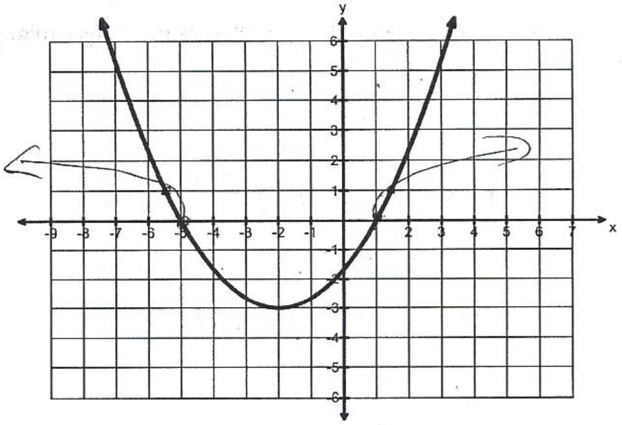
$= 18$

$f(x) = \sqrt{}$

$y = \sqrt{18}$



31. The graph of $y = f(x)$ is shown.



(A) On the same grid, sketch the graph of the function $y = \sqrt{f(x)}$ including all invariant points. (2 marks)

(1,0)   (-5,0)

(B) State the domain and range of $y = \sqrt{f(x)}$. (2 marks)

Domain: $x \le -5$, $x \ge 1$ OR $(-\infty, -5]$ or $[1, \infty)$

Range: $y \ge 0$

(C) State where the function $y = \sqrt{f(x)}$ is undefined and justify your reasoning. (1 mark)

$-5 < x < 1$, in this domain the y-values are negative and can't take square root of negative numbers.

32. Algebraically determine all the roots of $P(x) = x^3 - x^2 - 26x - 24$ (4 marks)

Possible roots: ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24.

$P(-1) = (-1)^3 - (-1)^2 - 26(-1) - 24 = 0$ ∴ $x = -1$ is a root

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -26 & -24 \\ & & -1 & 2 & 24 \\ \hline & 1 & -2 & -24 & \boxed{0} \end{array}$$

$x^2 - 2x - 24$

$(x+4)(x-6)$

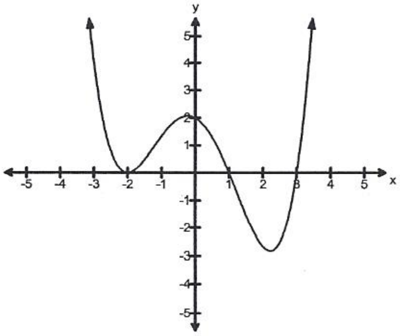
$x = -4$; $x = 6$

$\{x = -1, -4, 6\}$

33. Determine the equation of the polynomial function shown below. (4 marks)

$x = -2, 1, 3$

$m2$



$f(x) = a(x+2)^2(x-1)(x-3)$

$2 = a(0+2)^2(0-1)(0-3)$

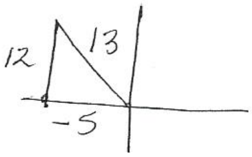
$2 = a(4)(-1)(-3)$

$\frac{2}{12} = \frac{12a}{12}$

$a = \frac{1}{6}$ $\therefore$ $f(x) = \frac{1}{6}(x+2)^2(x-1)(x-3)$

34. Given $cot\theta = -\frac{5}{12}$, $90° \leq \theta \leq 180°$, determine the values of $sin\theta$ and $sec\theta$.

(3 marks)



$sin\theta = \frac{12}{13}$

$sec\theta = \frac{1}{cos\theta} = -\frac{13}{5}$

35. Solve for θ: $4\csc^2\theta = -4\csc\theta + 15$, where $-\pi \leq \theta \leq 2\pi$. (5 marks)

$4csc^2\theta + 4csc\theta - 15 = 0$

$(2csc\theta - 3)(2csc\theta + 5) = 0$

$2csc\theta - 3 = 0$ $2csc\theta + 5 = 0$

$csc\theta = \frac{3}{2}$ $csc\theta = \frac{-5}{2}$

$\frac{1}{sin\theta} = \frac{3}{2}$ $\frac{1}{sin\theta} = -\frac{5}{2}$

$sin\theta = \frac{2}{3}$ $sin\theta = -\frac{2}{5}$

$\theta = \{0.73, 2.41, 3.55, 5.87, -0.41, -2.73$

$\theta = sin^{-1}(2/3)$ ref∠: $\theta = sin^{-1}(+\frac{2}{5})$

$\theta = 0.41$

$\theta = 0.73$

sin is neg in

Q3: $\pi + 0.41 = 3.55$

$3.55 - 2\pi = -2.73$

sin pos in

Q1: $\theta = 0.73$

Q2: $\theta = \pi - 0.73 = 2.41$

Q4: $2\pi - 0.41 = 5.87$

$5.87 - 2\pi = -0.41$