Math 3200: Ch. 3 Review - Polynomial Functions

1. $3x^3 - 2x^8 + 2x - 7$
   a) Degree = 8 (highest exp)
   b) linear term = $2x$
   c) leading coefficient = -2 (coefficient of highest power)

2. $y = 4x^3 - 2x + 6$

3. $f(x) = -(x+4)(x-1)^3$ (B)

4. cubic - (C)

5. Zeros: $x^2 = 0$ $3x - 2 = 0$ $x + 4 = 0$ (A)
   $x = 0$ $x = \frac{2}{3}$ $x = -4$

6. $R = 2(-2)^4 + 3(-2)^3 - 7(-2) - 8$ (C)
   (16, -8)
   $R = 32 - 24 + 14 - 8 = 14$

7. $h(-3) = 0$
   $K(-3)^2 + 2(-3) - 12 = 0$ (D)
   $9K - 6 - 12 = 0$
   $9K = 18$
   $K = 2$

8. $y = (x-3)(x+4)^2x^2$ (C)

9. $y = 4x^3 - 12x^2 + 8x$ (D)
   $0 = 4x(x^2 - 3x + 2)$
   $0 = 4x(x-2)(x-1)$
   $x = 0$ $x = 2$ $x = 1$

10. Degree = $3+2+1 = 6$ (D)

11. $f(-1) = R$

$= 2(-1)^{78} - 3(-1)^{9} + 4$ (D)

$= 2(+1) - 3(-1) + 4$

$= 2 + 3 + 4$

$= 9$

12. $P(x) = (x+1)(2x^2+x-1) - 4$

$= 2x^3 + x^2 - x + 2x^2 + x - 1 - 4$

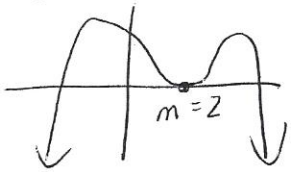
$P(x) = 2x^3 + 3x^2 - 5$

13. possible integral roots = factors of constant term 4

$\pm 1, \pm 2, \pm 4$ (B)

14. even degree, mult. = 2, neg. leading coefficient

∴ Q3 IQ4



(C)

15. max # turns = degree − 1 (C)

$= 6 - 1$

$= 5$

16. $P(x) = a(x+2)(x-4)^2$ (0, −3) point

$-3 = a(0+2)(0-4)^2$

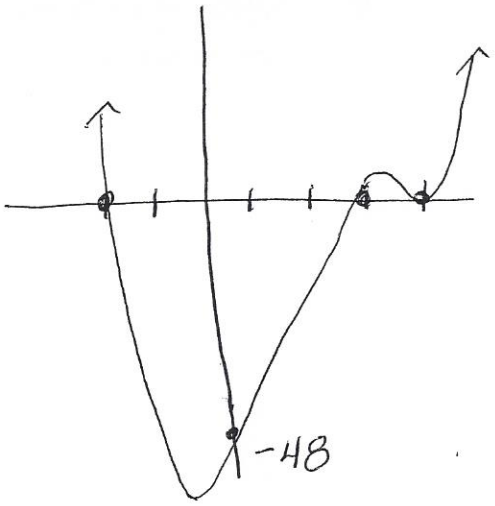
$-3 = a(2)(16)$

$\frac{-3}{32} = \frac{32a}{32}$

$-\frac{3}{32} = a$

$P(x) = -\frac{3}{32}(x+2)(x-4)^2$

17. $P(x) = \frac{1}{2}(x+2)(x-3)(x-4)^2$



x-ints: $x = -2, 3, 4 (m=2)$

y-int: $P(0) = \frac{1}{2}(0+2)(0-3)(0-4)^2$

$= \frac{1}{2}(2)(-3)(16)$

$= -48$

$a = \frac{1}{2}$ $a > 0$ ∴ Q1 Q2

Degree = 4 ∴ 'W'-shape

18. $(2x^4 - 5x^2 + 2x - 3) \div (x-2)$

2	2	0	−5	2	−3
		4	8	6	16
	2	4	3	8	13

Quotient: $2x^3 + 4x^2 + 3x + 8$ $R = 13$

19. $P(1) = P(-2) = P(4) = 0$ $P(2) = 16$ $(2, 16)$

x-ints: $x = 1, x = -2, x = 4$

factors: $x-1=0$ $x+2=0$ $x-4=0$

$P(x) = a(x-1)(x+2)(x-4)$

$16 = a(2-1)(2+2)(2-4)$

$16 = a(1)(4)(-2)$

$\frac{16}{-8} = \frac{-8a}{-8}$

$-2 = a$

$P(x) = -2(x-1)(x+2)(x-4)$

20. $(x+2)$ a factor $\therefore$ $x=-2$ is a root

$\therefore P(-2)=0$

$$\frac{K^2(-2)^3}{4} - K(-2)^2 + 3(-2) + 12 = 0$$

$$-2K^2 - 4K - 6 + 12 = 0$$

$$-2K^2 - 4K + 6 = 0$$

$$-2(K^2 + 2K - 3) = 0$$

$$-2(K+3)(K-1) = 0$$

$$K = -3 \quad K = 1$$

21. D

22. $V = 2x^3 - 5x^2 - x + 6$ (C)

$$\begin{array}{r|cccc} 2 & 2 & -5 & -1 & 6 \\ & & 4 & -2 & -6 \\ \hline & 2 & -1 & -3 & 0 \end{array}$$

$$2x^2 - x - 3 = 0$$

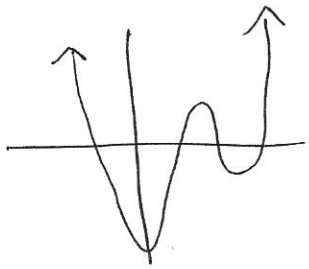
$$(2x-3)(x+1) = 0$$

23. $a\boxed{x^4} + bx^3 + cx^2 + dx + e$ (D)

$D = 4 \Rightarrow$ even degree

$a > 0$ positive $\therefore$ Q1 Q2

$e < 0$ negative $e = y\text{-int} = -(y\text{-int})$



24. $P(x) = 4x^3 - 3x^2 + Kx + 6$

$P(1) = P(-3)$

$4(1)^3 - 3(1)^2 + K(1) + 6 = 4(-3)^3 - 3(-3)^2 + K(-3) + 6$

$4 - 3 + K + 6 = -108 - 27 - 3K + 6$

$K + 7 = -3K - 129$

$+3K \quad -7 \qquad +3K \quad -7$

$\frac{4K}{4} = \frac{-136}{4}$

$K = -34$

25. $\frac{P(x)}{x-2} = x^2 - 4x + 6 + \frac{-7}{x-2}$

$P(x) = (x-2)(x^2 - 4x + 6) - 7$

$= x^3 - 4x^2 + 6x - 2x^2 + 8x - 12 - 7$

$P(x) = x^3 - 6x^2 + 14x - 19$

26. (C)

27. $P(x) = x^3 - x^2 - 14x + 24$

possible roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

2	1	−1	−14	24
		2	2	−12
	1	1	−12	0

$x^2 + x - 12 = 0$

$(x - 3)(x + 4) = 0$

Zeros: $x = 2 \quad x = 3 \quad x = -4$

28. $P(x) = -2x^4 - 10x^3 + 8x^2 + 40x$

i) end behaviour:

$D = 4$ (even)

$a = -2 \quad a < 0$ neg ∴ ↓↓

- extending from Q3 to Q4.

ii) $P(x) = -2x(x^3 + 5x^2 - 4x - 20)$

$P(x) = -2x((x^3 + 5x^2) - (4x + 20))$

$= -2x(x^2(x+5) - 4(x+5))$

$= -2x((x+5)(x^2-4))$

$P(x) = -2x(x+5)(x-2)(x+2)$

$x = 0, -5, 2, -2$

OR $P(x) = (-2x)(x^3 + 5x^2 - 4x - 20)$

$-2x = 0$

$x = 0$

2	1	5	−4	−20
		2	14	20
	1	7	10	0

$x^2 + 7x + 10 = 0$

$(x+5)(x+2) = 0$

$x = 0 \quad x = 2 \quad x = -5 \quad x = -2$

29. $P(x) = 4x^4 - 7x^3 + mx^2 + nx + 6$

$x-1$ is a factor $\therefore P(1) = 0$

divided by $(x+1)$, $R = 30$ $\therefore P(-1) = 30$

$P(1) = 0$

$4(1)^4 - 7(1)^3 + m(1)^2 + n(1) + 6 = 0$

$4 - 7 + m + n + 6 = 0$

$m + n + 3 = 0$ $\quad m + n = -3$ ①

$P(-1) = 30$

$4(-1)^4 - 7(-1)^3 + m(-1)^2 + n(-1) + 6 = 30$

$4 + 7 + m - n + 6 = 30$

$m - n + 17 = 30$

$-17 \quad -17$

$m - n = 13$ ②

$m + n = -3$

$m - n = 13$

$2m = 10$

$m = 5$

$m + n = -3$

$-m + n = -13$

$2n = -16$

$n = -8$

OR $m + n = -3$

$5 + n = -3$

$n = -8$