Mathematics 3200 Name: Key

Unit 6: Trigonometric Identities Date:

**Part A: Selected Response:** Place the letter of the correct response in the space provided (9 marks)

1. What are the non- permissible values for $x$ for the equation $\sec x \sin x = \tan x$? 1. D

$\frac{1}{\cos x} \cdot \sin x$

A) $x \neq 0 + \frac{\pi}{2}k, k \varepsilon I$

B) $x \neq 0 + \pi k, k \varepsilon I$

$\cos x = 0$

$\frac{\pi}{2} + \pi k$

C) $x \neq \frac{\pi}{2} + \frac{\pi}{2}k, k \varepsilon I$

D) $x \neq \frac{\pi}{2} + \pi k, k \varepsilon I$

(0,1)

2. What is the simplified form for $\cos\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} + x\right)$? 2. B

$\left(\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x\right) - \left(\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x\right)$

$(0)\quad (1)\qquad\qquad 0$

$\sin x - (-\sin x)$

$2\sin x$

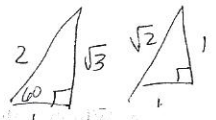
A) $-2\sin x$

B) $2\sin x$

C) 0

D) 2

3. What is the exact value of $\sin\frac{7\pi}{12}$? $105^\circ$ 3. A




$\sin(60+45)$

$= \sin 60 \cdot \cos 45 + \cos 60 \sin 45$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

A) $\frac{\sqrt{6}+\sqrt{2}}{4}$

B) $\frac{\sqrt{6}-\sqrt{2}}{4}$

C) $\frac{\sqrt{3}+\sqrt{2}}{2}$

D) $\frac{\sqrt{3}-\sqrt{2}}{2}$

4. Which is the simplified form of the trigonometric expression $\cot^2 x \sec x + \frac{1}{\cos x}$? 4. D

$\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos x} + \frac{1}{\cos x} \cdot \frac{\sin^2 x}{\sin^2 x}$

$\frac{\cos x}{\sin^2 x} \quad \frac{1}{\sin^2 x \cdot \cos x}$

A) $\sin^2 x \cos x$

B) $\sin^2 x + \cos x$

C) $\frac{1}{\sin^2 x} + \frac{1}{\cos x}$

D) $\frac{1}{\sin^2 x \cos x}$

5. Given that $\sin\theta = -\frac{4}{5}$, where $\pi \le \theta \le \frac{3\pi}{2}$, what is the exact value of $\cos 2\theta$? 5. A

A) $-\frac{7}{25}$ B) $-\frac{1}{25}$

C) $\frac{1}{25}$ D) $\frac{7}{25}$

$\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(-\frac{4}{5}\right)^2$

$= 1 - 2\left(\frac{16}{25}\right)$

$= 1 - \frac{32}{25}$

$= \frac{25}{25} - \frac{32}{25} = \frac{-7}{25}$

6. Solve: $\tan x \cos x = 1$ where $0 \le x \le 2\pi$? 6. B

A) $x = 0°$

B) $x = 90°$

C) $x = 180°$

D) no solution

$\frac{\sin x}{\cos x} \cdot \cos x = 1$

$\sin x = 1$

$x = 90°$

(-1,0) (0,1) (1,0) (0,-1)

7. Which is the simplified form of the expression $\csc x \tan x \cos^2 x$? 7. B

A) $\sin x$

B) $\cos x$

C) 1

D) $\cos^2 x$

$\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} \cdot \cos^2 x = \cos x$

8. Which is the simplified form of the trigonometric expression $\frac{\tan x - \tan x \cos^2 x}{\sin^3 x}$ 8. A

A) $\sec x$

B) $\frac{\cos x}{\sin^2 x}$

C) $\csc x$

D) $\frac{\sin^3 x}{\cos x}$

$= \frac{\tan x(1 - \cos^2 x)}{\sin^3 x}$

$= \frac{\tan x \sin^2 x}{\sin^3 x}$

$= \frac{\tan x}{\sin x} = \frac{\sin x}{\cos x} \times \frac{1}{\sin x} = \frac{1}{\cos x}$

$= \sec x$

9. Which is the simplified form of the trigonometric expression $\frac{\csc x - \sin x}{\cot^2 x}$? 9. D

A) $-\tan^2 x$

B) $\frac{1}{1 - \cos x}$

C) $\frac{\cos^4 x}{\sin^3 x}$

D) $\sin x$

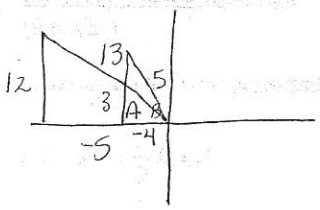
$\left[\frac{1}{\sin x} - \frac{\sin x}{1}\right] \div \frac{\cos^2 x}{\sin^2 x}$

$\frac{1 - \sin^2 x}{\sin x} \times \frac{\sin^2 x}{\cos^2 x}$

$\frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos^2 x} = \sin x$

**Part B: Constructed Response:** Show workings to receive full marks.

10. $\angle A$ and $\angle B$ are both in Quadrant II, $\cos A = -\frac{5}{13}$ and $\sin B = \frac{3}{5}$. Determine the exact value of $\cos(A+B)$. (4 marks)




$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \left(\frac{-5}{13}\right)\left(\frac{-4}{5}\right) - \left(\frac{12}{13}\right)\left(\frac{3}{5}\right)$

$= \frac{20}{65} - \frac{36}{65}$

$= -\frac{16}{65}$.

11. Verify the trigonometric identity: $\frac{\sin 2x}{1-\cos 2x} = \cot x$ $\frac{\cos x}{\sin x}$ (4 marks)

$LS = \frac{2\sin x \cdot \cos x}{1-(1-2\sin^2 x)} =$

$= \frac{2\sin x \cdot \cos x}{1-1+2\sin^2 x}$

$= \frac{2\sin x \cdot \cos x}{2\sin^2 x}$

$= \frac{\cos x}{\sin x}$ $LS = RS$.

$= \cot x = RS$

12. Solve: $3\cos 2x + \cos x = 2$ where $0 \le x \le 2\pi$ (4 marks)

$3(2\cos^2 x - 1) + \cos x = 2$

$6\cos^2 x - 3 + \cos x = 2$

$-2 \quad -2$

$6\cos^2 x + \cos x - 5 = 0$

$(6\cos x - 5)(\cos x + 1) = 0$

$6\cos x - 5 = 0$ $\quad \cos x + 1 = 0$

$\frac{6\cos x}{6} = \frac{5}{6}$ $\quad \cos x = -1$

$\cos x = \frac{5}{6}$ $\quad x = \pi$

$x = \cos^{-1}(5/6)$

$x = 0.59$ rad

cos pos in Q1 & Q4

$\therefore x = 0.59$ & $x = 2\pi - 0.59$

$= 5.69$

$x = 0.59, \pi, 5.69$