

## Lesson 8.4: Logarithmic and Exponential Equations

(i) If  $\log_c x = \log_c y$  then  $x = y$  where  $c > 0, c \neq 1$

(ii) If  $x = y$  then  $\log_c x = \log_c y$  where  $c > 0, c \neq 1$

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### Example 1

Solve  $2(25^{x+1}) = 250$

## Lesson 8.4 Logarithmic and Exponential Equations

### Example 2



Solve Logarithmic Equations and Verify

a)  $\log_3(\log_2 x) = 1$

b)  $\log_6(2x - 1) = \log_6 11$

c)  $\log(8x + 4) = 1 + \log(x + 1)$

d)  $\log_2(x + 3)^2 = 4$



## Lesson 8.4 Logarithmic and Exponential Equations

e)  $\log(x^2 + 12) = \log 8 + \log x$

f)  $\log x + \log(x - 5) = 2 \log 6$

g)  $\log_3(x^2 - 6x + 8) - \log_3(x - 4) = 1$

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**Your Turn**



h)  $\log_7 x + \log_7 4 = \log_7 12$

Answer:  $x = 3$

i)  $\log_2(x - 6) = 3 - \log_2(x - 4)$

Answer:  $x = 8$

j)  $\log_3(x^2 - 8x)^5 = 10$

Answer:  $x = -1, x = 9$



**Your Turn**



k)  $\log_5(x+1) + \log_5(x-2) = \log_5 4$

Answer:  $x = 3$

l)  $\log_2(4x-1) - \log_2(2x+1) = 3$

Answer:  $x = -\frac{3}{4}$

m)  $\log_2(x+6) + \log_2(x+4) = \log_2 8$

Answer: No solution

Questions: p.412-413 #1abcd, 3, 4bd, 5abcd, 6abc, 8a-e

## Lesson 8.4 Logarithmic and Exponential Equations

### Example 3

Solve Exponential Equations using Logarithms

Review:

a) Solve:  $4^{2x-1} = 8^{x+3}$

Solve:

b)  $4^x = 12$

c)  $8(3^{2x}) = 568$

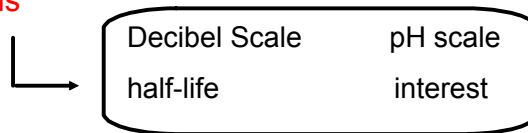
d)  $4^{2x-1} = 3^{x+2}$

e)  $6^{3x+1} = 8^{x+3}$

Questions: p.412-413 #2abcd, 7abcd

## Lesson 8.4 Logarithmic and Exponential Equations

### Application Questions



### Example 4

Sound intensity  $\beta$ , in decibels is defined as  $\beta = 10 \log\left(\frac{I}{I_0}\right)$  where  $I$  is the intensity of the sound measured in watts per square metre ( $\text{W/m}^2$ ) and  $I_0$  is  $10^{-12}$   $\text{W/m}^2$ , the threshold of hearing.

$$\beta = 10 \log\left(\frac{I}{I_0}\right) \quad \text{or} \quad \beta = 10(\log I + 12)$$

What is the sound intensity of a fire truck siren that has a decibel level of 122 dB?

Decibel Scale	
0 dB	Threshold for human hearing
10 dB	
20 dB	Whisper
30 dB	Quiet library
40 dB	Quiet conversation
50 dB	
60 dB	Normal conversation
70 dB	Hair dryer
80 dB	
90 dB	Lawnmower
100 dB	
110 dB	Car horn
120 dB	Rock concert
150 dB	Jet engine up close

For each increase of 10 on the decibel scale, there is a tenfold increase in the intensity of sound.



### Example 5

Audiologists recommend that people should wear hearing protection if the sound level exceeds 85 dB.

The sound level of a chain saw is about 85dB.

The maximum volume setting on a media player (with headphones) is about 110dB.

How many times as intense is the maximum volume setting on a media player compared to the chainsaw?





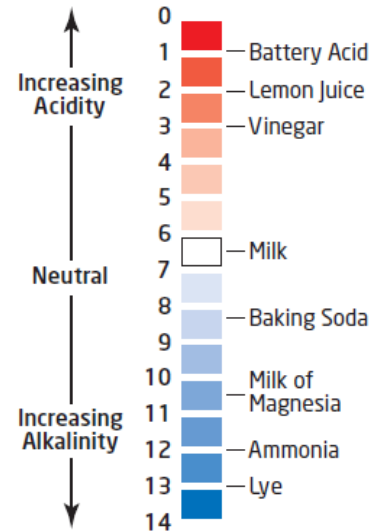
**Example 6**

The pH scale is used to measure the acidity of a solution.

The pH is defined as  $p = -\log x$

$$pH = -\log[H^+]$$

where  $[H^+]$  is the hydrogen ion concentration in moles per litre



a) A common ingredient in cola drinks is phosphoric acid, the same ingredient found in many rust removers. A cola drink has a pH of 2.5. Milk has a pH of 6.6.

How many times acidic is a cola drink compared to milk?

b) An apple is 5 times as acidic as a pear. If a pear has a pH of 3.8, then what is the pH of an apple?



### Example 7

A radioactive sample has a half life of 3 years. If the initial mass of the sample is 67g, how long will it take for the sample to reach 7g?

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

### Example 8

When an animal dies, the amount of radioactive carbon-14 in its bone decreases. Archaeologists use this fact to determine the age of a fossil based on the amount of C-14 remaining. The half life of C-14 is 5730 years. If a buffalo skull had 49.5% of C-14 left, how old were the bones when they were found?



Buffalo skull display, Head-Smashed-In buffalo Jump Visitor Centre, near Fort McLeod, Alberta



## Lesson 8.4 Logarithmic and Exponential Equations

### Interest Review: Simple vs Compound Interest

Simple Interest  $\longrightarrow$  interest is always based on original principal

Compound Interest  $\longrightarrow$  interest is added to the principal

**Example:** \$1000 earning 5% per year

Simple	Compound
1st year:	
2nd year:	

$$A = A_0(1+r)^n$$

### Compounding Periods

Annually: Once per year

Semi-Annually: Twice per year

Quarterly: 4 times per year

Monthly: 12 times per year

Daily: 365 times per year

$\longrightarrow$

### Example 9

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Money is invested at 6% per annum interest. How long will it take the investment to double if the interest is compounded monthly.

$$A = A_0(1 + r)^n$$

### Example 10

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Christina plans to buy a car. She has saved \$5000. The car she wants costs \$5900. How long will Christina have to invest her money in a term deposit that pays 6.12% per year, compounded quarterly, before she has enough to buy the car?

$$A = A_0(1 + r)^n$$



### Example 11



One ATV cost \$14 000 and depreciates in value at 6% every 3 years, while another ATV costs \$9200 and depreciates in value at 5% every 2 years. If both ATV's were purchased at the same time, when will the value of both be the same?

