

Section 7.3: Solving Exponential Functions

Review: Exponent Properties

A. Negative Exponents

$$b^{-x} = \frac{1}{b^x}$$

a) 3^{-2}

b) x^{-4}

c) $\left(\frac{3}{4}\right)^{-2}$

d) $\frac{1}{6^{-2}}$

B. Zero Exponent

$$b^0 = 1$$

a) $\left(\frac{2}{3}\right)^0$

b) $5x^0$

c) $(5x)^0$

C. Multiplication Rule

$$(b^x)(b^y) = b^{x+y}$$

a) $2^2 \times 2^3$

b) $x^4 \cdot x^2$

c) $5x^2y^4(3x^3y^2)$

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D. Quotient Rule

$$\frac{b^x}{b^y} = b^{x-y}$$

a) $\frac{x^5}{x^3}$

b) $\frac{12x^2}{4x^{-3}}$

c) $\frac{16x^3y^7}{8x^5y^4}$

E. Power Rule

$$(b^x)^y = b^{xy}$$

a) $(2^5)^2$

b) $(2x^{-2})^3$

c) $\left(\frac{1}{3x^5}\right)^{-2}$

F. Rational Exponents

$$b^{\frac{1}{n}} = \sqrt[n]{b} \quad \text{and} \quad b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

a) $9^{\frac{3}{2}}$

b) $64^{\frac{2}{3}}$

c) $\left[\left(\frac{16}{9}\right)^{-3}\right]^{\frac{1}{2}}$

d) $\frac{\left(\frac{1}{2}\right)^{-3} - 27^{\frac{2}{3}} - 5^0}{4^{\frac{1}{2}}}$

→

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G. Common Base Rule: $b^x = b^y$ if and only if $x = y$

a) $2^x = 2^3$

b) $5^x = 25^7$

c) $2^{x+3} = 2^4$

d) $27^{x+5} = 81$

H. Common Exponent Rule: $a^x = b^x$ if and only if $a = b$

a) $2^5 = x^5$

b) $\left(\frac{1}{2}\right)^4 = r^4$

c) $\left((3)^2\right)^6 = \left((x)^4\right)^3$

d) $x^{-7} = \left(\frac{1}{4}\right)^7$

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Your Turn

1. Express each of the following as a power with a base of 2.

a) 8 b) $\frac{1}{16}$ c) 8^{-2} d) $8^{\frac{2}{3}}(\sqrt{16})^3$

2. Express each of the following as a power with a base of 3.

a) 27 b) $\left(\frac{1}{9}\right)^4$ c) $27^{\frac{1}{3}}(\sqrt[3]{81})^2$

3. Express 64 as a power with a base of:

a) 2 b) 4 c) 8

4. Simplify each expression by rewriting each power with a common base

a) $\frac{4^3(16)^5}{8^6}$ b) $\frac{(81)^{-2}(9)^5}{(27)^2}$ c) $\frac{(8x^2)^{-3}(32x^3)^4}{(4x^{-2})^5}$



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Example 1: 

Write the following expression with a single base.

$$(a) \frac{(25^{3-2x})(5^{3x-7})(125^{x+1})}{25^{x+2}}$$

$$(b) \frac{(2^x)(8^{x+2})(32^{2x-1})}{(8^x)(\sqrt{4^x})}$$

$$(c) \frac{(\sqrt[3]{8^{3x}})(\sqrt{4})^{x-3}(16^{3x+1})}{(\sqrt[5]{32^2})^{x+1}(4^{x+2})}$$



Example 2:

Solve an Equation by Changing the Base

(a) $2^{3x} = 64$

(b) $4(3^{x+2}) = 36$

(c) $8^{3x-4} + 7 = 71$

(d) $4^{2x} = 8^{2x-3}$

(e) $9^{4x} = 27^{x-1}$

(f) $\frac{8^{3x+1}}{16^{4(2x\pi-1)}} = 4^{5x-1}$

→

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Your Turn

Algebraically determine the solution for each of the following equations:

$$(a) 5^x = \frac{1}{125}$$

$$(b) \left(\frac{1}{4}\right)^x = 16$$

$$(c) 3^{2x+1} = 3^{x+2}$$

$$(d) 4^{3x+5} = 2^{4x-3}$$

$$(e) 27^{-x+3} = 81^{2x+2}$$

$$(f) \left(\frac{1}{32}\right)^{x-2} = \left(\frac{1}{4}\right)^{5x-3}$$

$$(g) 2(3^{x+2}) = 18$$

$$(h) 9(2^{3x+5}) - 8 = 28$$

→

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$$(i) \frac{25^x}{5^{x+3}} = \left(\frac{1}{125}\right)^{x+1}$$

$$(j) \frac{(3^{x+2})(27^x)}{81^{x-3}} = \left(\frac{1}{27}\right)^{-x}$$

$$(k) 27^{2x-1} = \sqrt[3]{3}$$

$$(l) \sqrt{3^x} = 9^{2x+1}$$

$$(m) \sqrt[5]{8^{x-1}} = \left(\sqrt[3]{16}\right)^x$$

$$(n) \sqrt{5} = 25^{x-1}$$

→

Example 3



Given $f(x) = 2(3^{-x+3})$ algebraically determine the value of x if $f(x) = \frac{2}{9}$



Applications

↳ half-life $A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$

interest $A = A_0(1+r)^n$

Half life: amount decays to half its amount over a certain amount of time

Example 4

The half life of Radon 222 is 92 hours. From an initial sample of 48g, how long would it take to decay to 6g?

$$A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

→

Example 5

A bacteria culture starts with 4 000 bacteria and doubles every 0.5 hours. After how many hours will the bacteria count be 32 000.

Example 6

Mary invests \$500 and the value of the investment increases by 4% annually.

$$A = A_0(1 + r)^n$$

- a) How much money is in Mary's investment after 30 years?
- b) What amount of time will it take for the investment to double?

Different compounding periods will be discussed in the next unit

└─→ compounding semi annually
 compounding quarterly

Your Turn

A. The half life of a certain isotope is 30 hours. Determine the amount of time it takes for a sample of 1792 mg to decay to 56 mg?

B. If a new car, purchased for \$20 000 depreciates at a rate of 28% every two years

i) What will be the value of the car after 6 years?

ii) What amount of time will it take for the car to lose half its value?

Practice Questions P.364 #3, 4, 5, 9, 10, 11ab