

CHAPTER 2

Exponential Functions

Exponential Function: a function of the form $y = c^x$
where c is a constant ($c > 0$) and x is a variable

i) $c > 1$

ii) $0 < c < 1$

$\Rightarrow c$ affects the shape

Section 7.1: Characteristics of Exponential Function

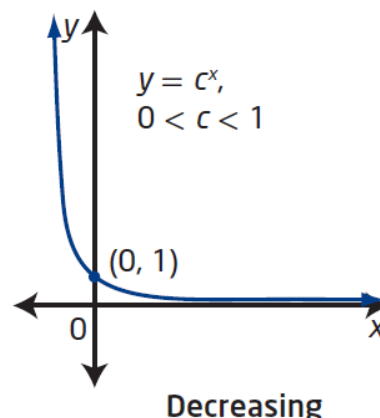
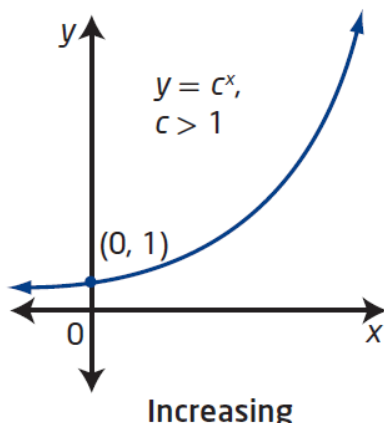
↳ graph of an exponential function $y = c^x$

→ increasing where $c > 1$

Example:

→ decreasing where $0 < c < 1$

Example:



Lesson 7.1 Characteristics of Exponential Functions

$$y = c^x$$

What will happen if $c = 1$?

↳ graph is constant
(does not increase or decrease)

x	-2	-1	0
y			

What will happen if $c = 0$?

↳ function is only defined
for positive real numbers.

x	1	2	3
y			

What will happen if $c < 0$?

↳ rational values of x result
in non- real values of y ;
discontinuous graph

$$y = (-4)^x$$

x	-2	0	$\frac{1}{2}$
y			

Lesson 7.1 Characteristics of Exponential Functions

Example 1

Graph the exponential function and identify the following:

(i) Domain and Range

a) $y = 4^x$

(ii) x-intercept and y-intercept

(iii) Increasing or Decreasing Function

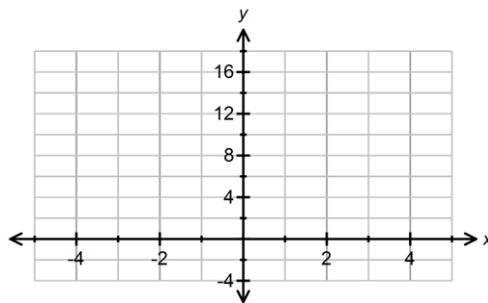
b) $y = \left(\frac{1}{2}\right)^x$

(iv) Equation of the horizontal Asymptote

Solution

a) $y = 4^x$

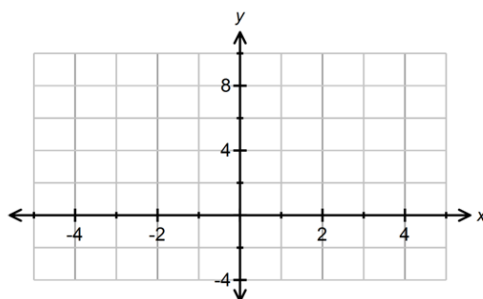
x	y



x-intercept _____
 y-intercept _____
 increasing/decreasing _____
 Horizontal Asymptote _____
 Domain _____
 Range _____

b) $y = \left(\frac{1}{2}\right)^x$

x	y



x-intercept _____
 y-intercept _____
 increasing/decreasing _____
 Horizontal Asymptote _____
 Domain _____
 Range _____

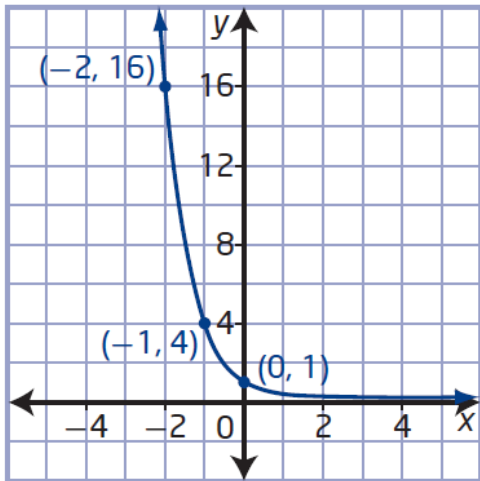


Example 2

Write the Exponential Function Given Its Graph

What function of the form $y = c^x$ can be used to describe the graph shown?

(a)

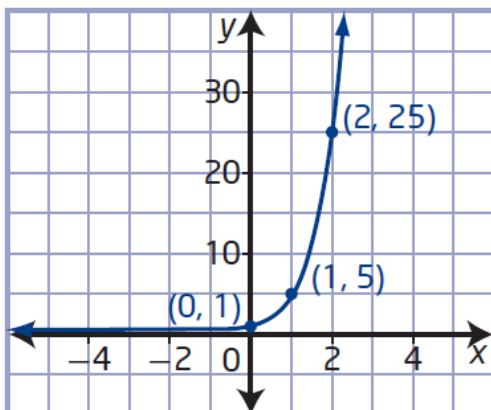


Hint: look for a pattern in the ordered pairs from the graph

x	y

Note: common ratio when $HA = 0$

(b)



x	y

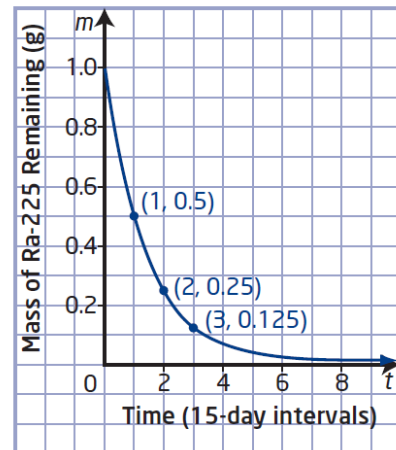
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Lesson 7.1 Characteristics of Exponential Functions

Example 3

Application of an Exponential Function

A radioactive sample of radium (Ra-225) has a **half life** of 15 days. The mass, m , in grams, of Ra-225 remaining over time t , in 15 day intervals, can be modelled using the exponential graph shown.



- What is the initial mass of Ra-224 in the sample?
- What value does the mass of Ra-225 approach?
- What is the domain and range of this function?
- Write the decay model that relates the mass of Ra-225 remaining, in 15 day intervals
- Estimate how many days it would take for Ra-225 to decay to $\frac{1}{30}$ of its original mass.

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Lesson 7.1 Characteristics of Exponential Functions

Summary:

exponential growth

- an increasing pattern of values that can be modelled by a function of the form $y = c^x$, where $c > 1$

exponential decay

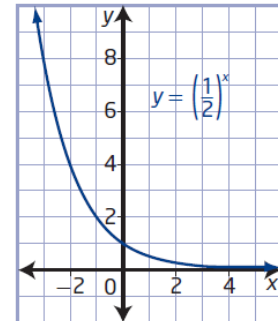
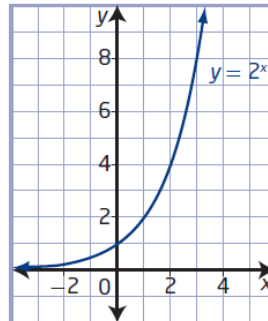
- a decreasing pattern of values that can be modelled by a function of the form $y = c^x$, where $0 < c < 1$

half-life

- the length of time for an unstable element to spontaneously decay to one half its original mass

Key Ideas

- An exponential function of the form $y = c^x$, $c > 0$,
 - is increasing for $c > 1$
 - is decreasing for $0 < c < 1$
 - is neither increasing nor decreasing for $c = 1$
 - has a domain of $\{x \mid x \in \mathbb{R}\}$
 - has a range of $\{y \mid y > 0, y \in \mathbb{R}\}$
 - has a y-intercept of 1
 - has no x-intercept
 - has a horizontal asymptote at $y = 0$



Practice Questions P.342-345 #3, 4, 6, 7

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