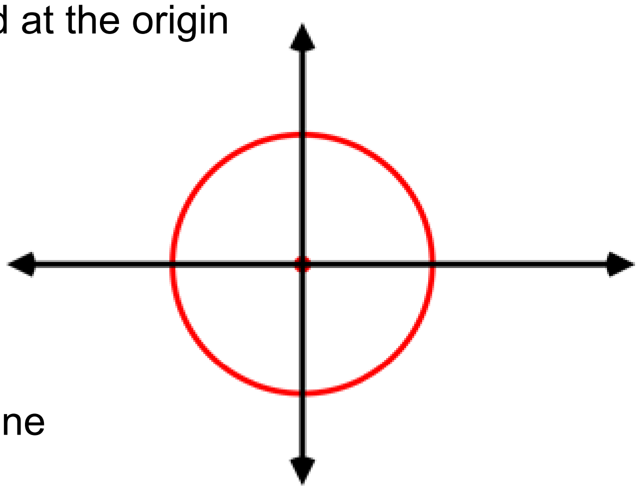
Lesson 4.2: The Unit Circle

- equation of a circle with centre (0,0) and radius r
- coordinates of points on a unit circle

Unit Circle: a circle with a radius of 1 unit.

Example 1:

Choose a point on the circle centered at the origin where the radius is 1.



Use Pythagorean theorem to determine the equation of the unit circle

a) What is the equation of the circle if $r = 2$?

b) What is the equation of the circle if $r = \sqrt{5}$?

Example 2:

Determine the equation of a circle with centre at the origin and radius 6.
Is the point (-2, 4) a point on the circle?

Example 3:

a) Determine the equation of a circle with centre at the origin and radius $\sqrt{34}$?

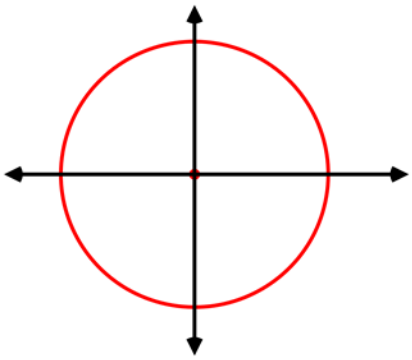
b) Determine the equation of a circle with centre at the origin and radius $2\sqrt{5}$?

→

Determine the coordinates for points on the unit circle

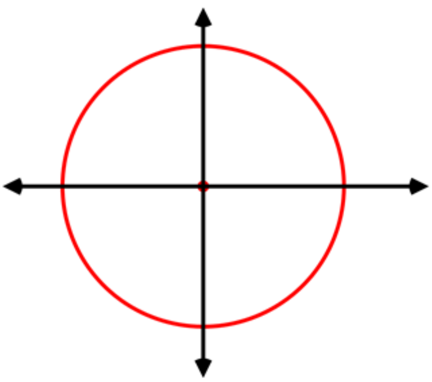
Example 4:

Determine the y-coordinate on the unit circle given the x-coordinate is $\frac{2}{3}$.



Example 5:

Determine the x-coordinate on the unit circle given the y-coordinate is and the point $-\frac{1}{\sqrt{2}}$ is in quadrant III .

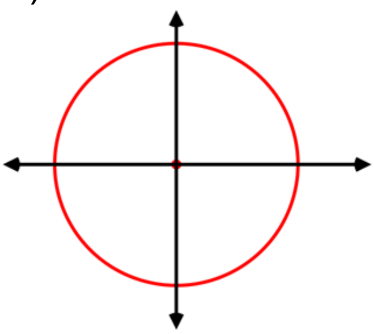


Your Turn

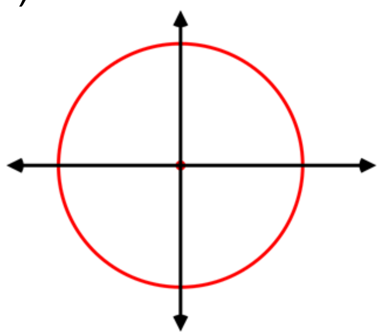
Determine the missing coordinate(s) for all points on the unit circle satisfying the given conditions. Draw a diagram and tell which quadrant(s) the points lie in.

a) $\left(-\frac{5}{8}, y\right)$ **b)** $\left(x, \frac{5}{13}\right)$, where the point is in quadrant II

a)



b)

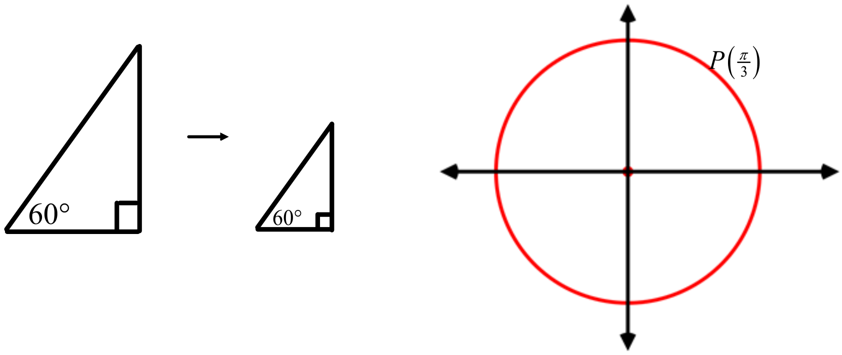


Assign p.186-187 #1ad, 2ace, 3bcef

Trigonometric Ratios to Coordinates of Points on a Unit Circle

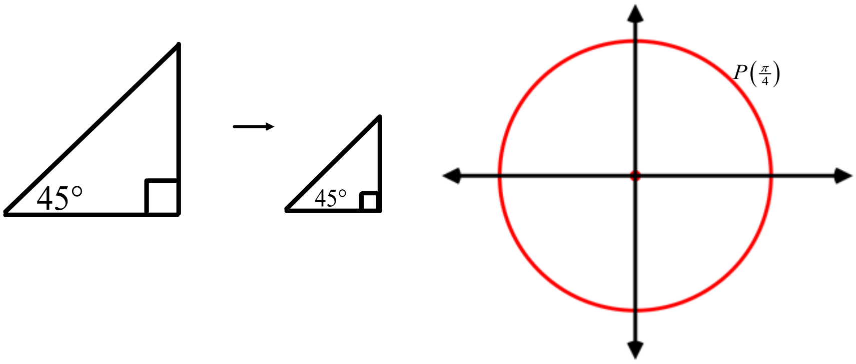
Mathematics 2200

$\cos 60°$



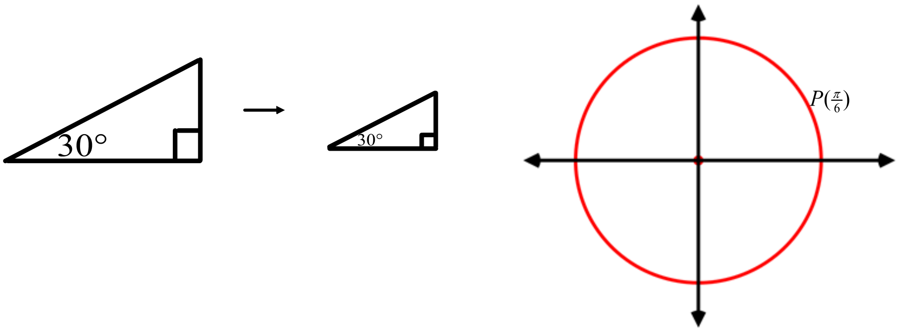
$\sin 60°$

$\cos 45°$



$\sin 45°$

$\cos 30°$



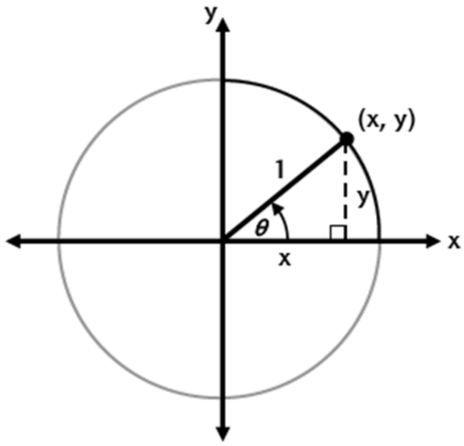
$\sin 30°$

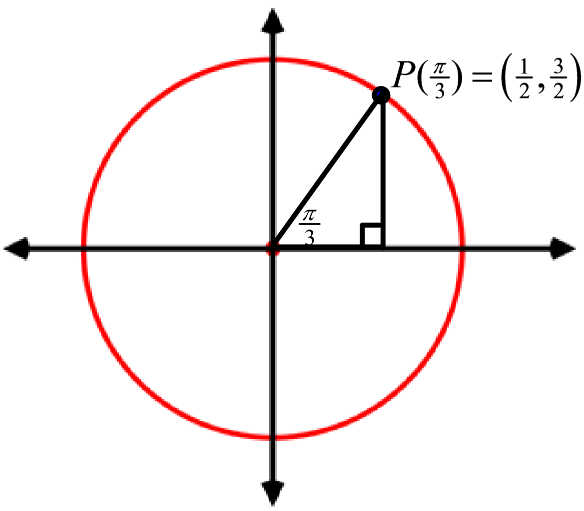
Relationship between trigonometric ratios to the coordinates of points on the unit circle

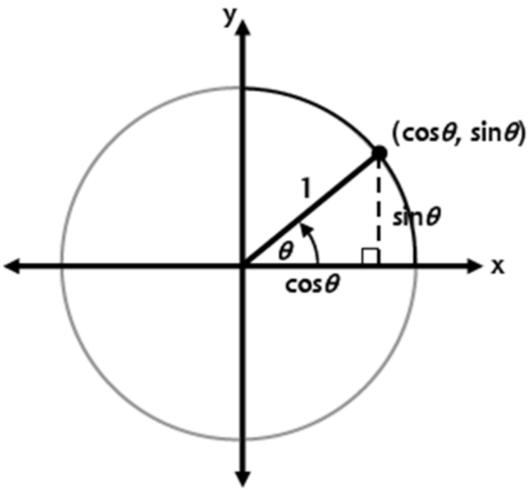
$$\cos\theta = \frac{adjacent}{hypotenuse} \longrightarrow$$

$$\sin\theta = \frac{opposite}{hypotenuse} \longrightarrow$$

$$\tan\theta = \frac{opposite}{adjacent} \longrightarrow$$



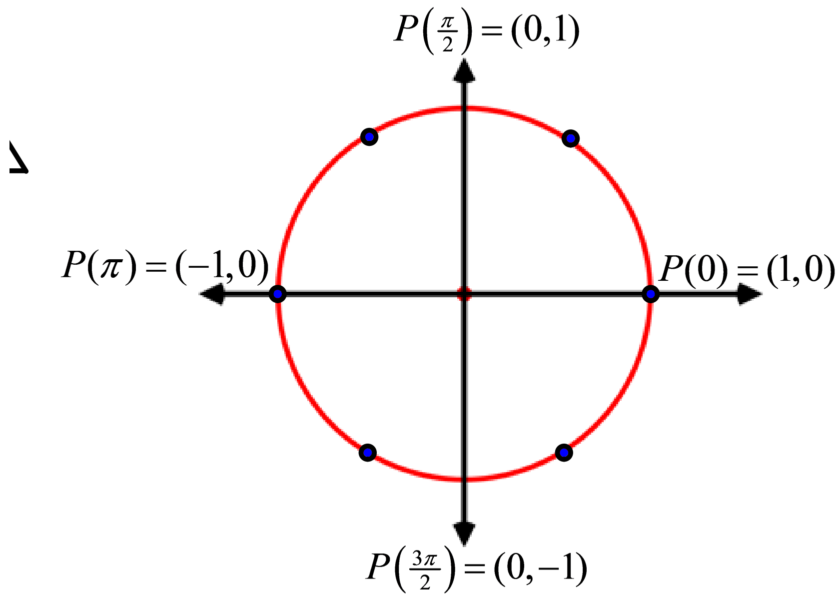


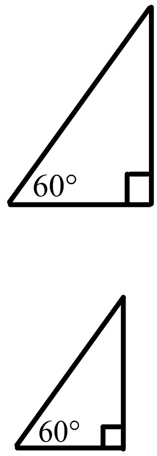


Special Angles $\frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{4}$

Some points $P(\theta)$ on the unit circle correspond to exact values of the special angles and their multiples.

Part 1: Multiples of $\frac{\pi}{3}$ or $60°$ on the unit circle





The points corresponding to angles that are multiples of $\frac{\pi}{3}$ that cannot be simplified have the same coordinates except for their signs

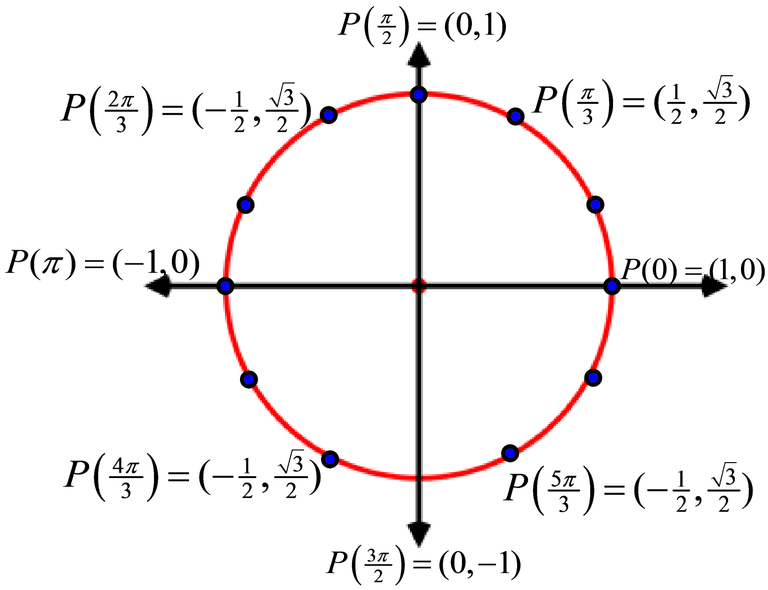
$P(\frac{\pi}{3}) =$

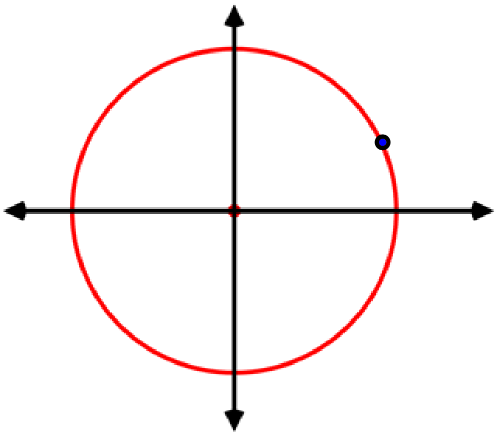
$P(\frac{4\pi}{3}) =$

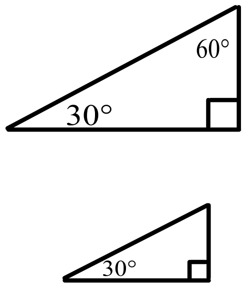
$P(\frac{2\pi}{3}) =$

$P(\frac{5\pi}{3}) =$

Part 2: Multiples of $\frac{\pi}{6}$ or $30°$ on the unit circle





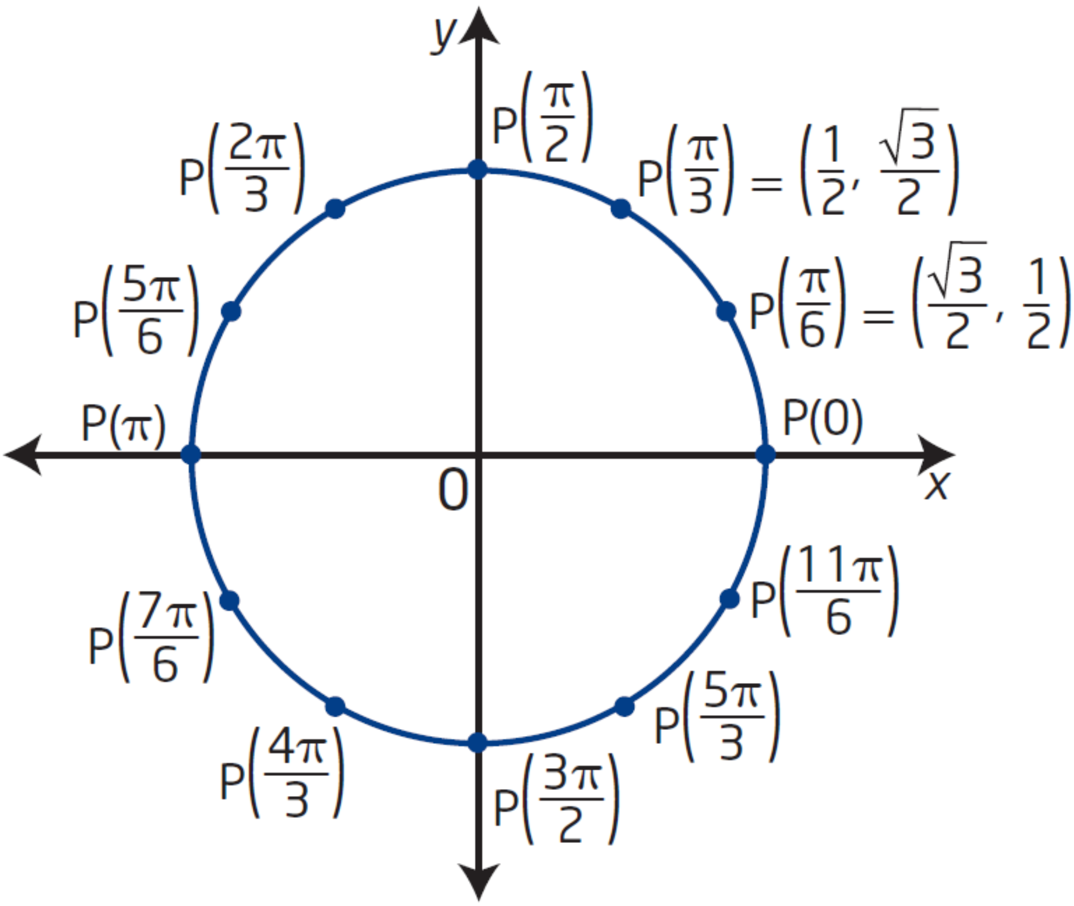


→ The points corresponding to angles that are multiples of $\frac{\pi}{6}$ that cannot be simplified have the same coordinates except for their signs

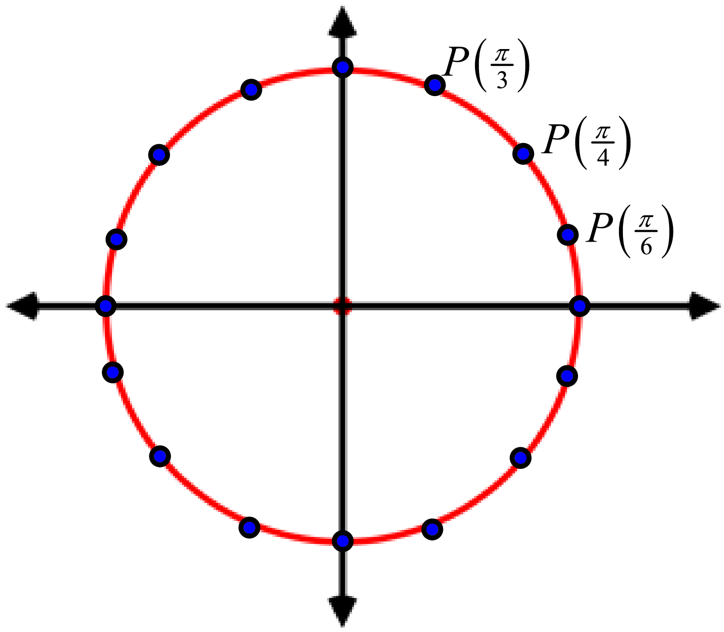
$P(\frac{\pi}{6}) =$ $\qquad$ $P(\frac{7\pi}{6}) =$

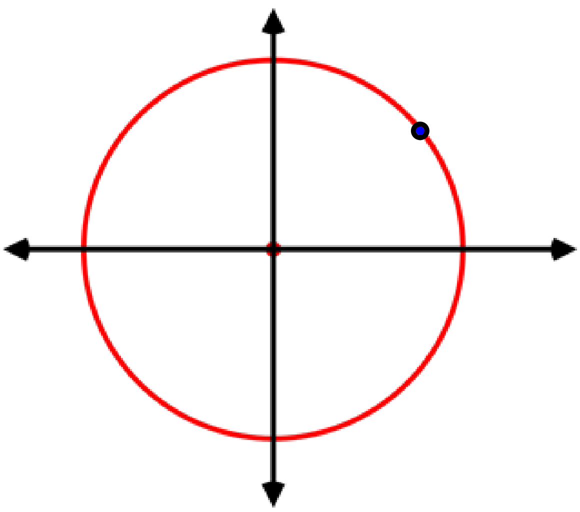
$P(\frac{5\pi}{6}) =$ $\qquad$ $P(\frac{11\pi}{6}) =$

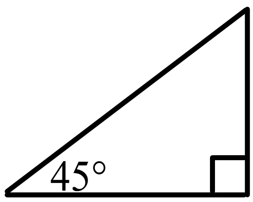
Summary of the unit circle with $\frac{\pi}{6}$ and $\frac{\pi}{3}$

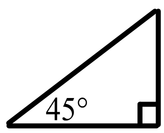


Part 3: Multiples of $\frac{\pi}{4}$ or $45°$ on the unit circle









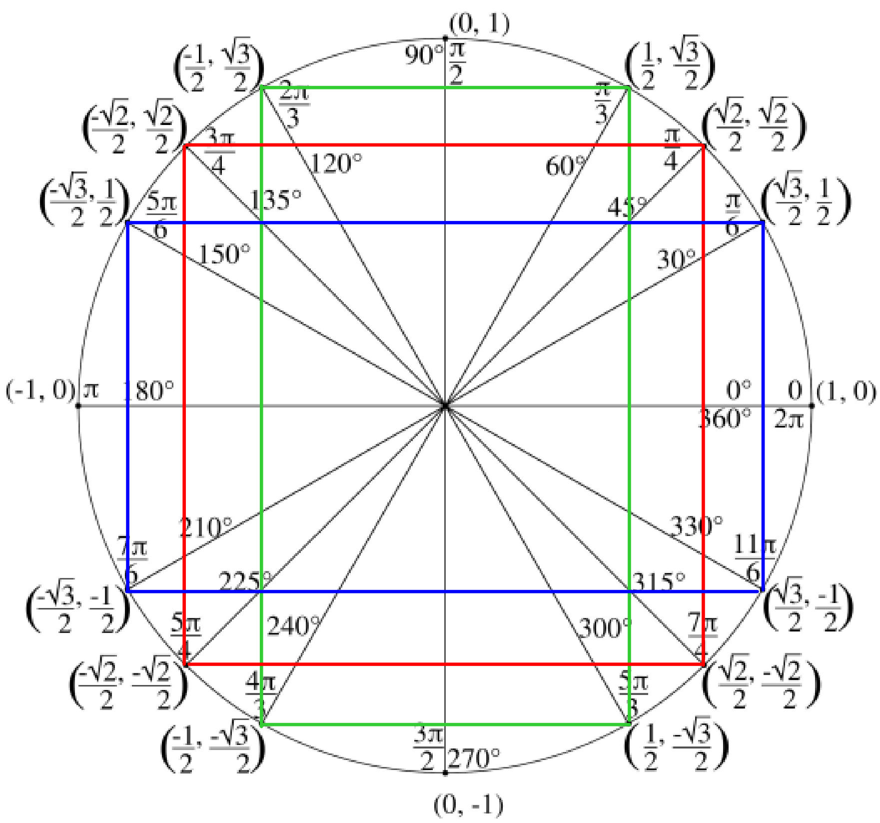
$P(\frac{\pi}{4}) =$

$P(\frac{5\pi}{4}) =$

$P(\frac{3\pi}{4}) =$

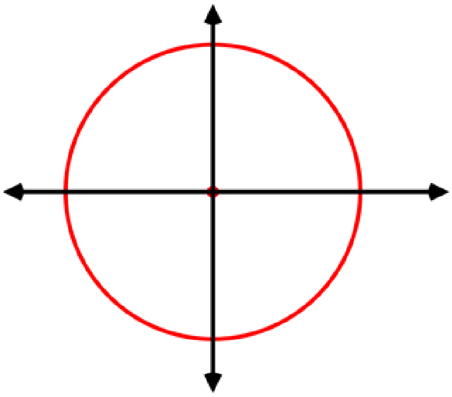
$P(\frac{7\pi}{4}) =$

The Unit Circle



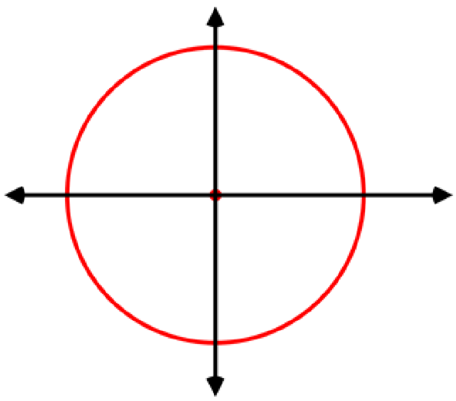
Example: p. 187 #5e

Identify a measure for the central angle θ where $0 \le \theta \le 2\pi$ such that $P(\theta)$ is $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



Example: p.187 #6

Determine one positive and one negative measure for θ if $P(\theta) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



Assign p.187 #4, 5