

CHAPTER 4

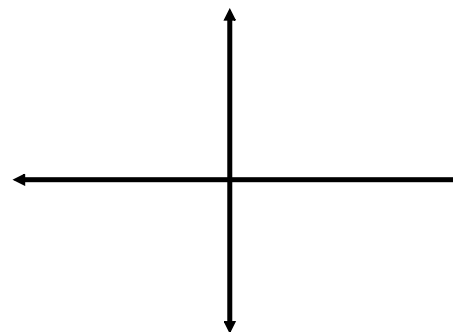
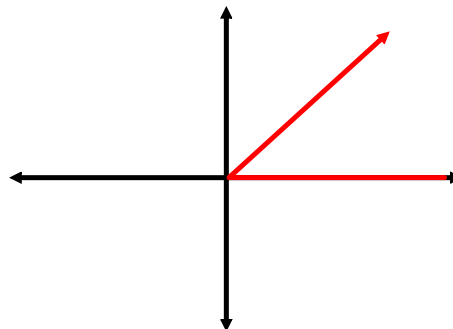
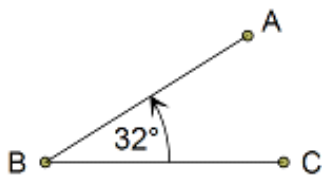
Trigonometry and The Unit Circle

Lesson 4.1: Angles and Angle Measure

(A) Standard Position

When drawing an angle θ on the x-y axis in standard position, the following conditions must apply:

- its vertex is at the _____
- its initial arm lies along the _____

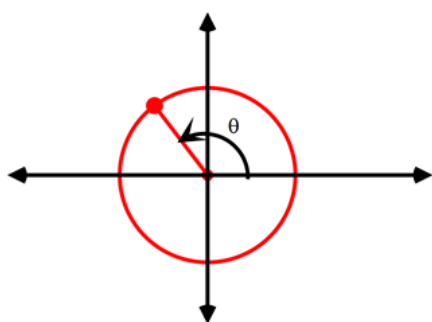


Lesson 4.1 Angles and Angle Measure

(B) Positive and Negative Rotation (Standard Position)

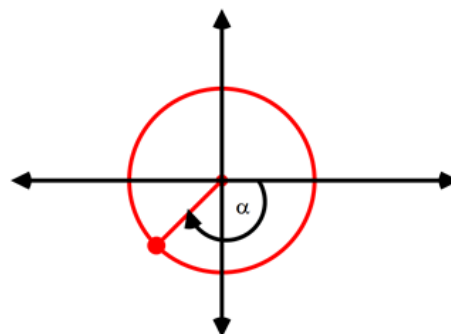
Positive angles:

counterclockwise rotation



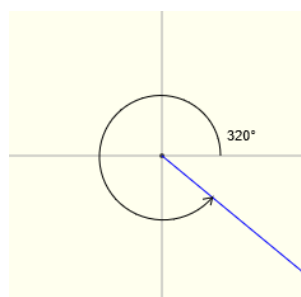
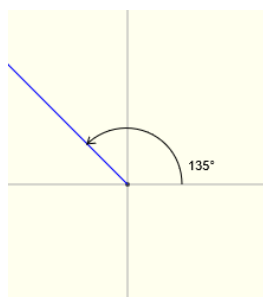
Negative angles:

clockwise rotation



(C) Reference Angle

↳ angle between the terminal arm and the x-axis



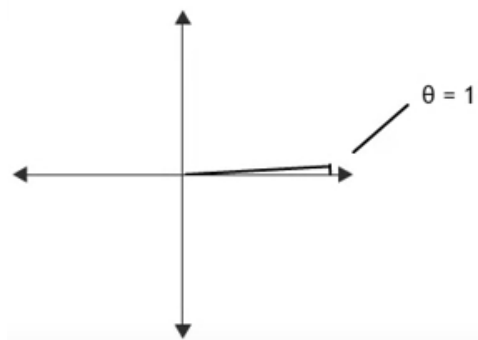
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Angle Measure

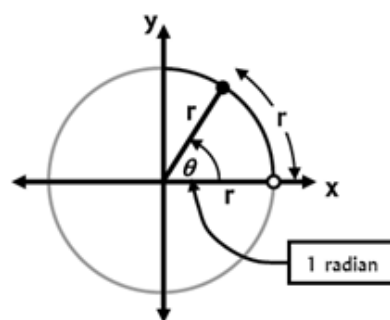
There are two units for measuring angles:

- » degrees
- » radians

One degree \longrightarrow is defined as $\frac{1}{360}$ of a full rotation



One radian \longrightarrow is the measure of the angle formed when the arc length of a circle has the same length as the radius.



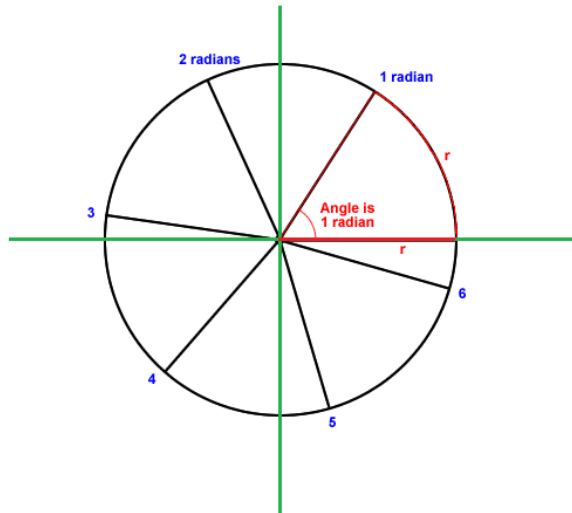
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Lesson 4.1 Angles and Angle Measure

Example 1: What if the arclength and the radius of the circle are equal?

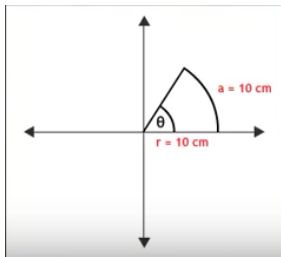


What is the measure of 1 radian in degrees?



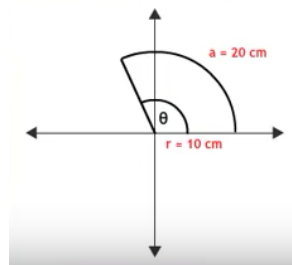
When the arc length is equal to the radius, we have an angle of 1 rad.

$$\theta = \frac{10 \text{ cm}}{10 \text{ cm}} = 1 \text{ rad } (57.3^\circ)$$



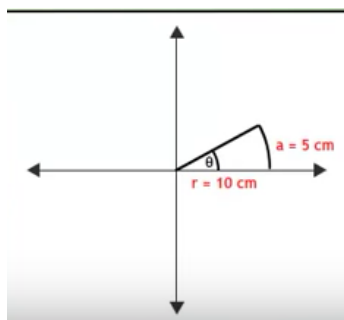
When the arc length is double the radius length, we have an angle of 2 rad.

$$\theta = \frac{20 \text{ cm}}{10 \text{ cm}} = 2 \text{ rad } (114.6^\circ)$$



When the arc length is half the radius length, we have an angle of 0.5 rad.

$$\theta = \frac{5 \text{ cm}}{10 \text{ cm}} = 0.5 \text{ rad } (28.65^\circ)$$



Lesson 4.1 Angles and Angle Measure

Example 2: What are some common radian and degree measures?

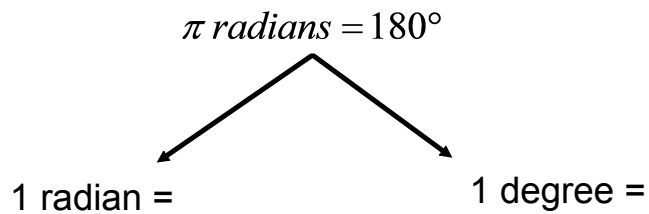
$$2\pi \text{ radians} = 360^\circ$$

$\pi \text{ rad}$	$\frac{\pi}{3} \text{ rad}$	$\frac{2\pi}{3} \text{ rad}$
$\frac{\pi}{2} \text{ rad}$	$\frac{\pi}{6} \text{ rad}$	
$\frac{\pi}{4} \text{ rad}$	$\frac{3\pi}{4} \text{ rad}$	

→

Lesson 4.1 Angles and Angle Measure

Convert From Radians to Degrees and vice versa



Formula:

To convert from radians to degrees	To convert from degrees to radians
$\text{radians} \times \frac{180}{\pi}$	$\text{degrees} \times \frac{\pi}{180}$

Example 3: Convert the following to radian measure (exact and approximate).

(i) 135°

(ii) -58°

(iii) 225°

(iv) 144°

(v) 214.5°

Lesson 4.1 Angles and Angle Measure

Example 4: Convert the following to degree measure.

(i) $\frac{2\pi}{3}$

(ii) $-\frac{7\pi}{4}$

(iii) $\frac{11\pi}{12}$

(iv) -5

(v) $\frac{4}{5}\pi$

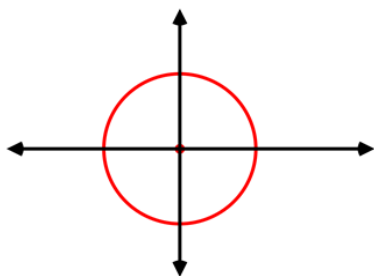
Since π radians = 180° ,
as a shortcut to convert
we can simply replace
 π with 180° .

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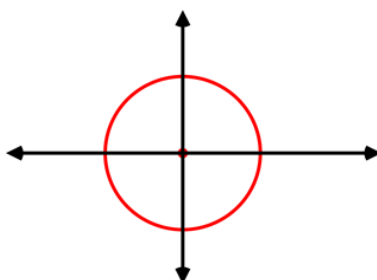
Lesson 4.1 Angles and Angle Measure

Example 5: Sketch the following angles in standard position.

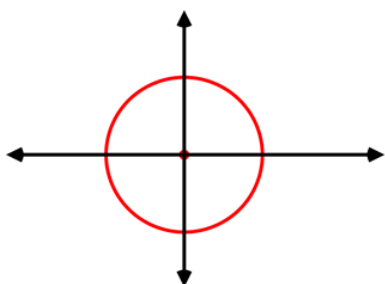
(i) $\frac{\pi}{4}$



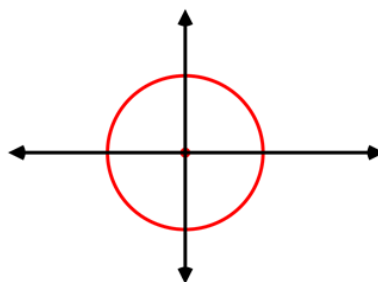
(ii) $-\frac{2\pi}{3}$



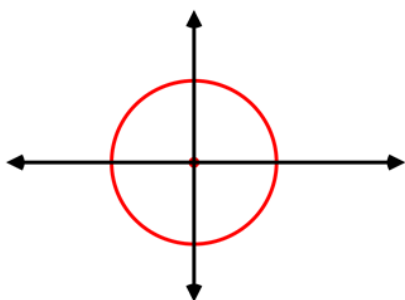
(iii) $\frac{7\pi}{6}$



(iv) -225°



(v) 2.57

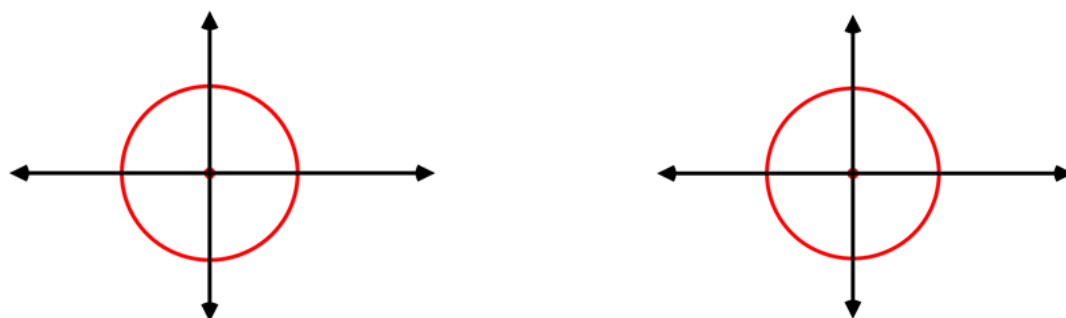


P175 #1, 3, 4, 5abd, 6

Coterminal Angles

└→ angles in standard position with the same terminal arm and can be measured in degrees or radians

Example 1: Show that 60° and -300° are coterminal angles.



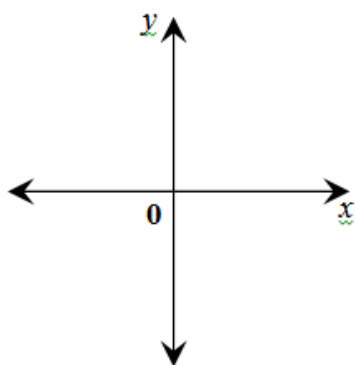
Note: Coterminal angles can be found by adding or subtracting multiples of 360° or 2π

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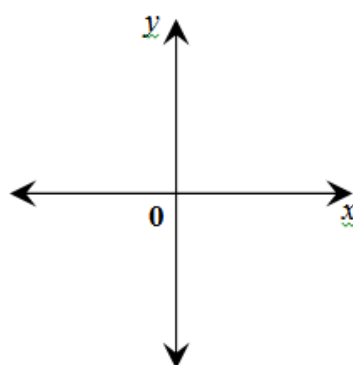
Lesson 4.1 Angles and Angle Measure

Example 2: Name one positive and one negative angle measure that is coterminal with each angle.

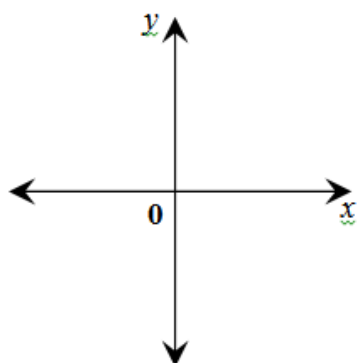
(i) 40°



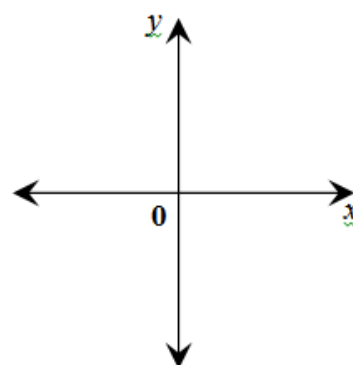
(ii) 310°



(iii) -515°



(iv) $\frac{3\pi}{4}$



Lesson 4.1 Angles and Angle Measure

Your Turn: Name one positive and one negative angle measure that is coterminal with each angle.

(i) 270°

(ii) -740°

(iii) $-\frac{5\pi}{4}$

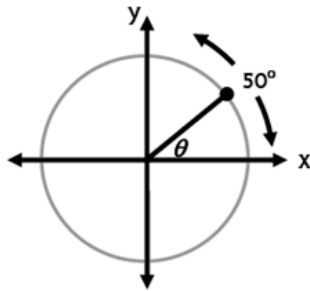
(iv) $\frac{3\pi}{8}$

→

Coterminal Angles in General Form

- Is it possible to list all of the possible coterminal angles?
- It is possible to write a formula to represent the group of angles in our solution.

Example 1:



Coterminal angles in CCW (+) direction

Coterminal angles in CW (-) direction

$\theta =$

$\theta =$

Combine the solutions in both directions to list ALL possible coterminal angles (in degree and radian measure)

General Form:

→

Lesson 4.1 Angles and Angle Measure

Example 2:

- (i) Express the angles coterminal with 110° in general form.
- (ii) Identify the angles coterminal with 110° that satisfy the domain
 $-720^\circ \leq \theta \leq 720^\circ$

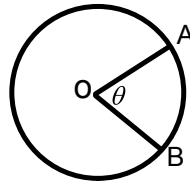
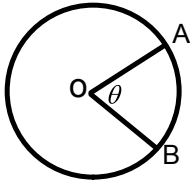
Example 3:

- (i) Write an expression for all possible angles coterminal with $\frac{8\pi}{3}$.
- (ii) Identify the angles that are coterminal that satisfy $-4\pi \leq \theta \leq 4\pi$.

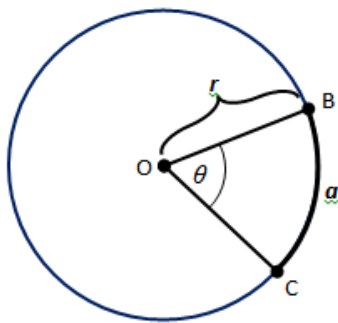
P.176 #7, 8, 9, 11abdf

Arc Length of a Circle

↳ minor arc/major arc



Determine a formula relating the radius (r), central angle (θ) and arc length of a circle (a).



a, r measured
in same units

↑
measured
in radians

$$a = \theta r$$

know how to
rearrange!

→

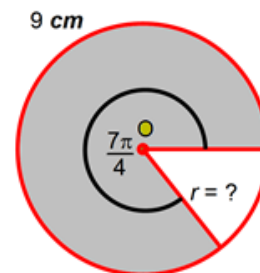
Lesson 4.1 Angles and Angle Measure

Example 1: Determine the measures of the arc length subtended by the angles and radii below:

(i) central angle of $\frac{2\pi}{3}$ with radius of 10cm.

(ii) central angle of 2.6 rad with radius of 4.9cm.

Example 2: Determine the measure of the radius of a circle in the following diagram.



Lesson 4.1 Angles and Angle Measure

Example 3:

Determine the missing quantities where a represents the length of the arc of a circle with radius r , subtended by a central angle of θ .

(i) $r = 8.7\text{cm}$, $\theta = 75^\circ$, $a = ?\text{ cm}$

(ii) $r = ?\text{ cm}$, $\theta = 1.8$, $a = 4.7\text{ mm}$

(iii) $r = 5\text{m}$, $\theta = ?$, $a = 13\text{ m}$

P.176 #12ac, 13

Lesson 4.1 Angles and Angle Measure

Attachments

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