Lesson 3.4 Graph and Equation of Polynomial Functions

Part A: Graph of a Polynomial Function

the x-intercepts of the graph
the zeros of the function
the roots of the equation } → **Multiplicity (of a zero)**

- A factor that is repeated.

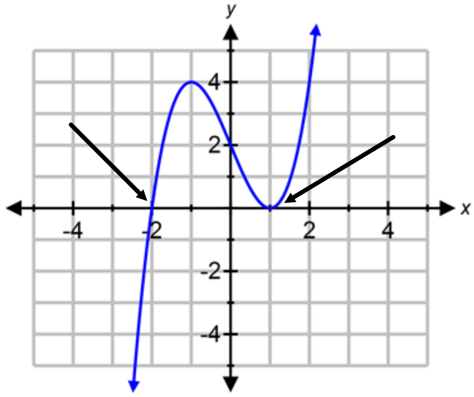
(also called the "order" of a zero)

Example 1

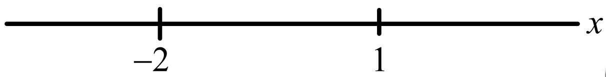
Consider the graph of the function $y = (x-1)^2(x+2)$

$x = 1$ ⟶

$x = -2$ ⟶



a) The x-intercepts divides the x-axis into <u>three</u> intervals.



(b) Complete the table to show where the function is **positive** (above the x-axis) or **negative** (below the x-axis) for each interval.

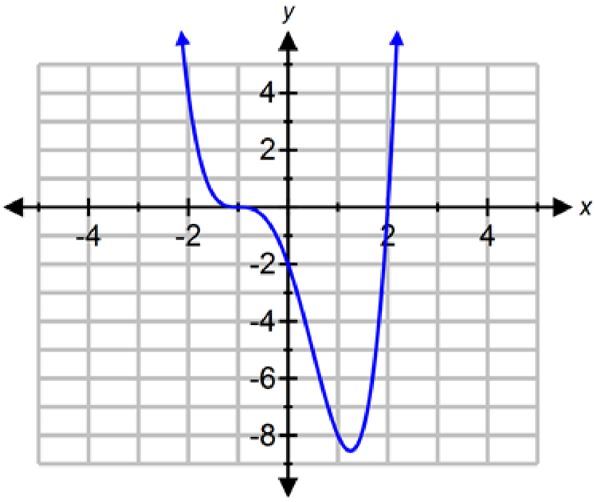
	positive	negative
set notation		
interval notation		

(c) What effect does a single root have on the graph?

(d) What effect does a double root have on the graph?

Example 2

Consider the graph of the function $P(x) = (x+1)^3(x-2)$



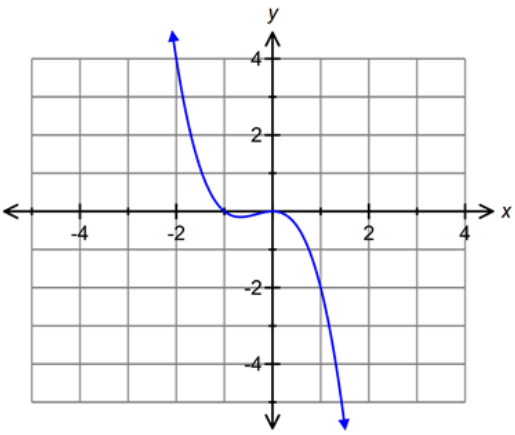
Zeros/Multiplicity

Positive Interval:

Negative Interval:

Example 3

Consider the function $P(x) = -x^2(x+1)$



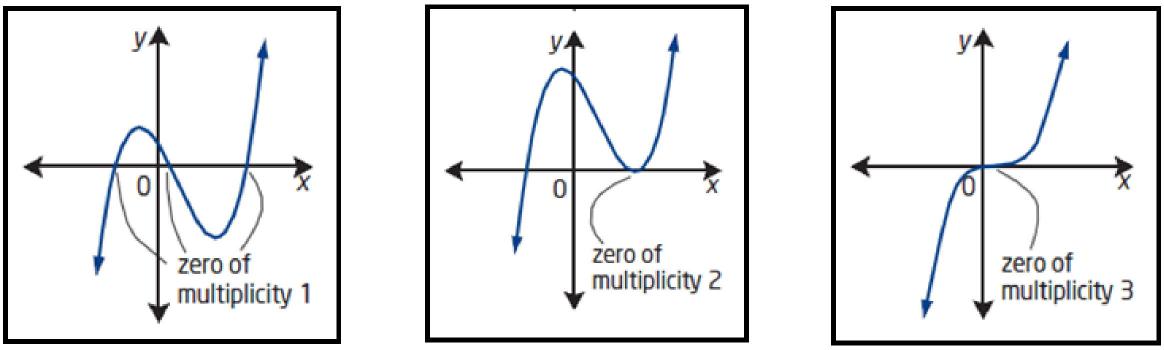
Zeros/Multiplicity:

Positive Interval:

Negative Interval:

Think About

The shape of the graph of a function close to a zero depends on its multiplicity.
A cubic function, for instance, can appear to have different shapes as seen below:



Note:

a zero of odd multiplicity, the sign of the function ____________

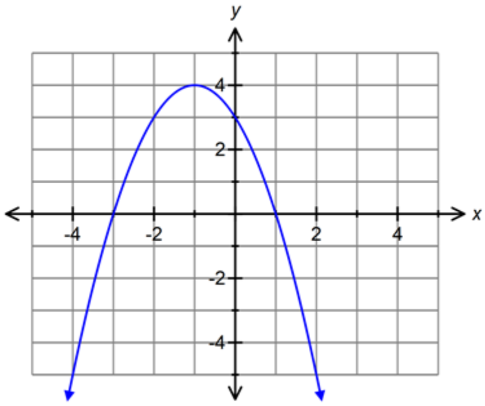
a zero of even multiplicity, the sign of the function _____________

Your Turn 1

(a) For the following graphs, determine the zero(s) and state the multiplicity.

(b) State the intervals where the function is positive and the intervals where it is negative

(i) $P(x) = -(x+3)(x-1)$

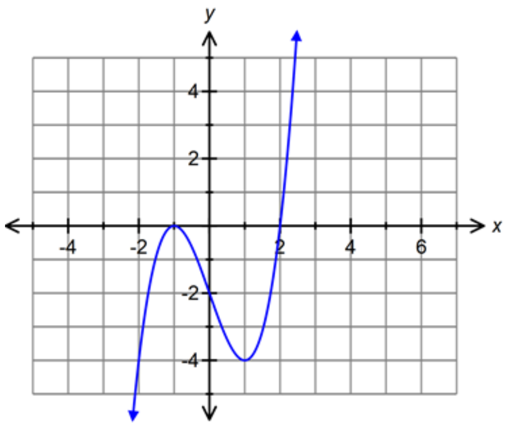


zeros:

positive interval:

negative interval:

(ii) $P(x) = (x+1)^2(x-2)$

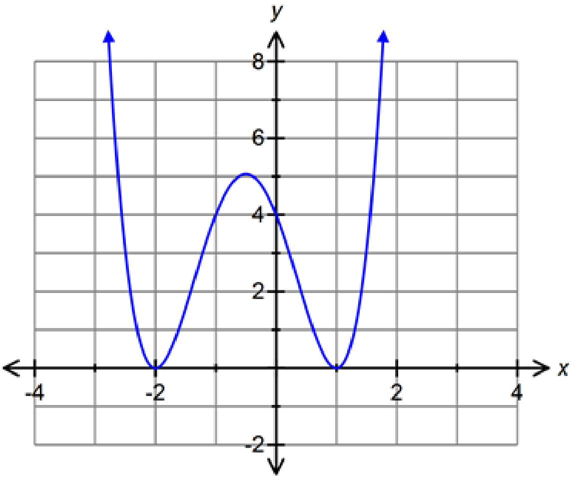


zeros:

positive interval:

negative interval:

(iii) $P(x)=(x-1)^2(x+2)^2$

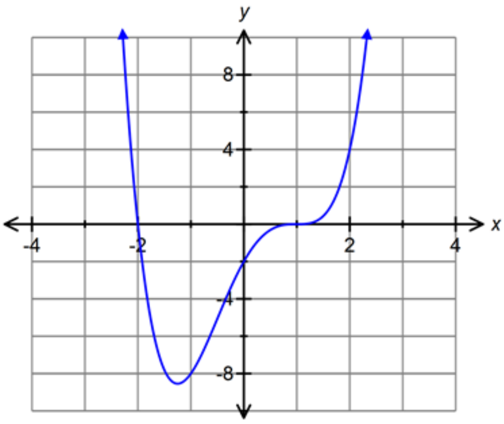


zeros:

positive interval:

negative interval:

(iv) $P(x)=(x-1)^3(x+2)$



zeros:

positive interval:

negative interval:

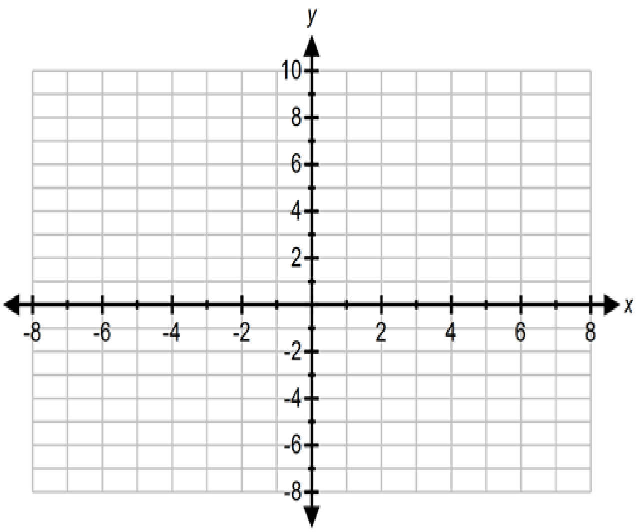
Textbook Questions: pg. 148 # 4abc

Sketch the Graphs of Polynomial Functions using intercepts and multiplicity of zeros

Example 4

Sketch the graph of each function. Label all intercepts.

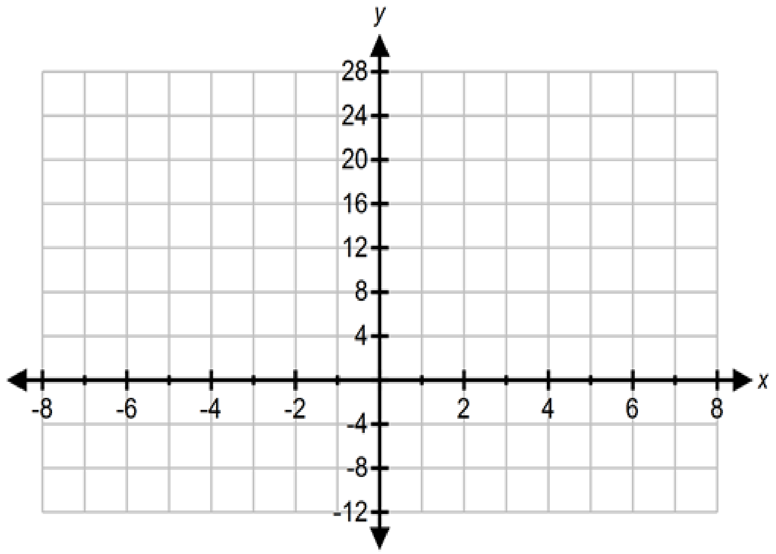
(*i*) $f(x) = -x^3 - 5x^2 - 3x + 9$



Example 5

Sketch the graph of each function. Label all intercepts.

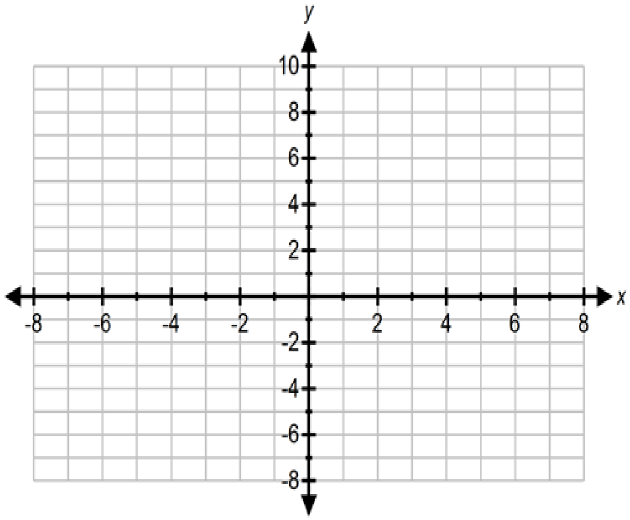
$(ii)\quad f(x) = -(x+1)^3(x-3)$



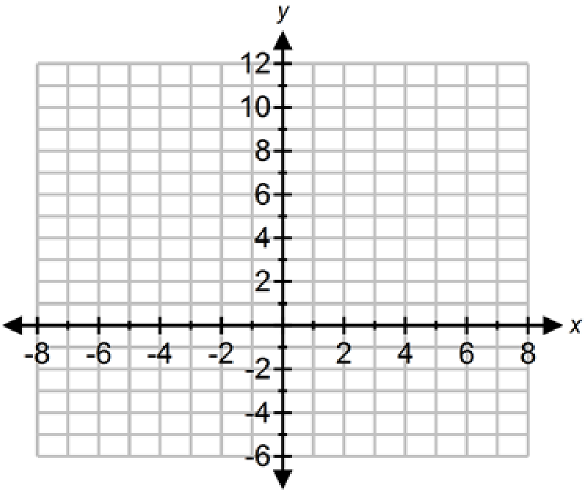
Your Turn 2

Sketch the graph of each function. Label all intercepts.

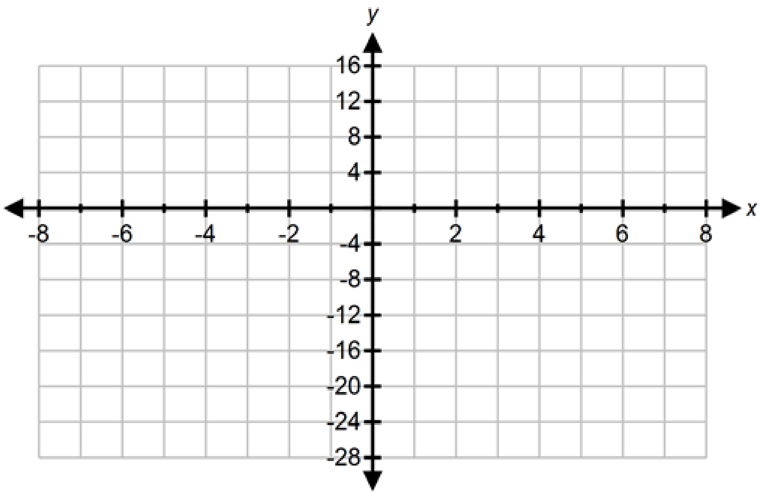
(iii) $f(x) = x^3 - 2x^2 - 4x + 8$



(iv) $f(x) = (x-1)^2(x+2)^3$



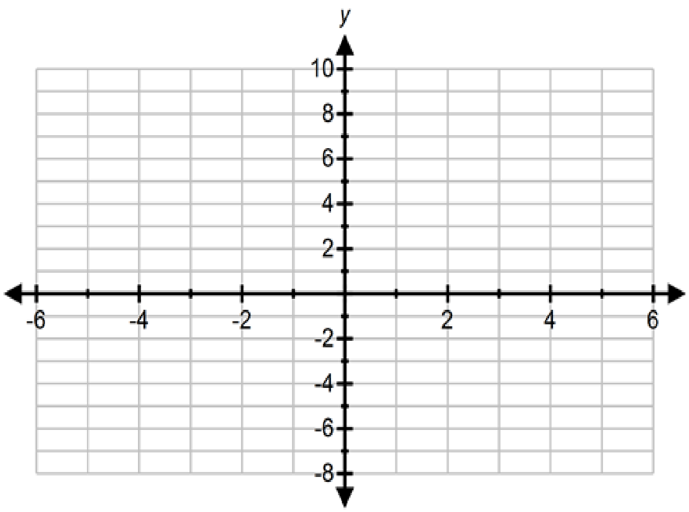
(v) $f(x) = -(x+1)(x+2)(x+3)(x-2)^2$



Your Turn 3

Draw the graph of a polynomial with the following characteristics:

- ✔ x-intercepts: (-1,0) and (3,0)
- ✔ sign of the leading coefficient: positive
- ✔ polynomial degree: 4
- ✔ relative maximum at (1,8)



Textbook Questions: pg. 147-149 #1ab, 2ac, 7cd, 8cd, 9ef

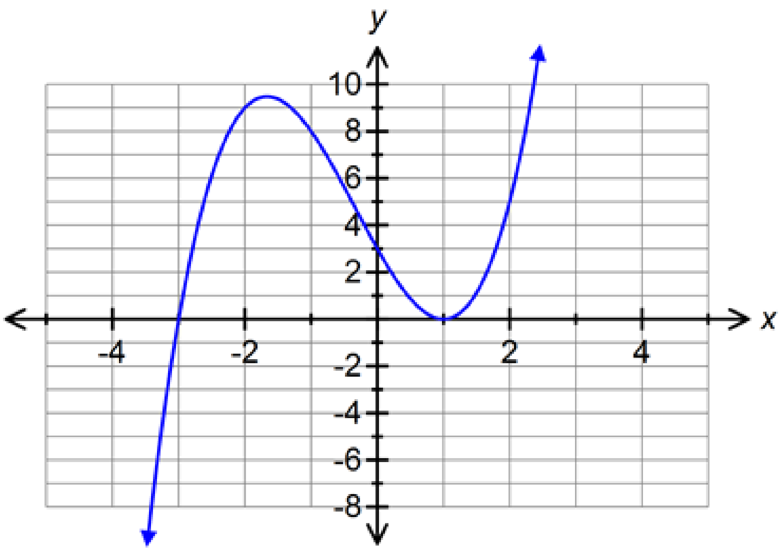
Part B: Equation of a Polynomial Given its Graph

Reminder: Identify the following characteristics:

- factors
- x-intercepts
- multiplicity of zeros
- sign and value of the leading coefficient
- y-intercept

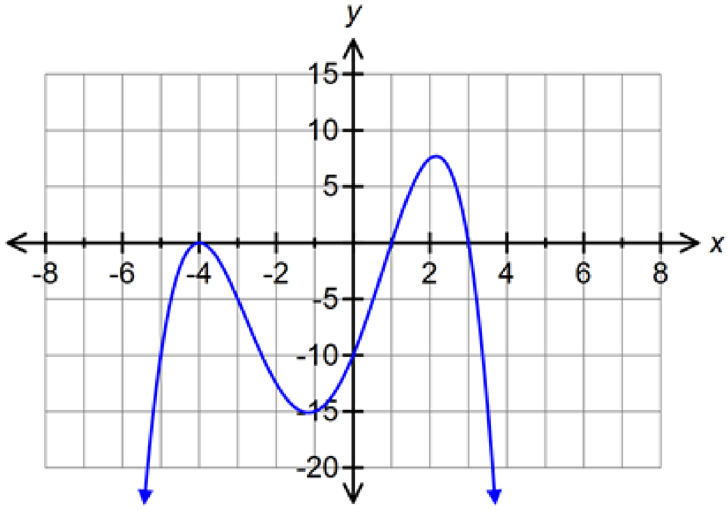
Example 6

Determine the equation of the polynomial function that corresponds to the graph:



Your Turn 4

Determine the equation of the polynomial function that corresponds to the graph:



Example 7

Determine the equation for each polynomial function:

(a) a cubic function with zeros -2 (multiplicity 2) and 4 and has a y-intercept -12.

(b) a quintic function with zeros -1 (multiplicity 3) and 4 (multiplicity 2) and has a constant term of 6.

(c) a quartic function with a negative leading coefficient, zeros -3 (multiplicity 2) and 3 (multiplicity 2) and has a y-intercept of -5.

Textbook Questions: pg.149 #10abcd, pg.150 #14abc

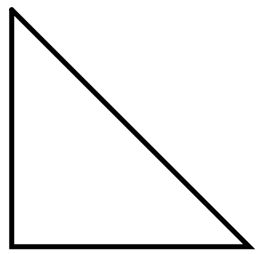
Part C: Solve Problems by Modeling a Situation with a Polynomial Function

→ Create an equation
Factor
Solve

Review: Using Quadratic Functions

Example 8

One leg of a right triangle exceeds the other leg by four inches. The hypotenuse is 20 inches. Find the length of the shorter leg of the right triangle.



Example 9

Three consecutive odd integers have a product of -315. What are the integers?

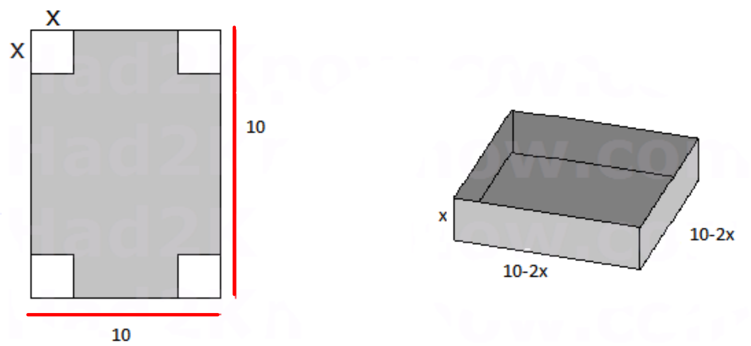
Example 10

The length, width and height of a rectangular box are x cm, $(x-4)$cm, and $(x+5)$ cm respectively. Determine the dimensions of the box if the volume is 132 cm^2

Is it necessary to place restrictions on the independent variable?

Example 11

An open topped box with a volume of 72 in^3 is made from a square piece of cardboard by cutting equal squares from each corner and folding up the sides. If the original dimension of the cardboard is 10 in., find the side length that is cut from each corner.



Example 12

The actual and projected number, C (in millions), of computers sold for the region between 2010 and 2020 can be modelled by

$$C = 0.0092(t^3 + 8t^2 + 40t + 400)$$

where $t = 0$ represents 2010. During which year are 8.51 million computers projected to be sold?

Textbook Questions pg. 150-151 #12, 13, 15-19