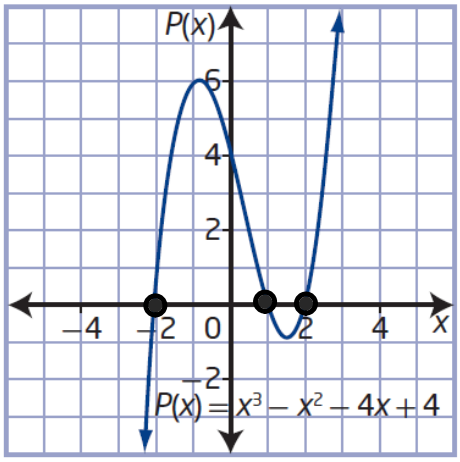
Lesson 3.3: The Factor Theorem

The zeros of a polynomial function or the x-intercepts of the graph are related to the factors of the polynomial.

x-intercepts	factors
x = -2	x+2
x = 1	x-1
x = 2	x-2



Factor Theorem

A polynomial $P(x)$ has a factor $x-a$ if and only if $P(a)=0$

Example 1

Given $P(x)=x^3-3x^2-4x+12$

i) Is $x-1$ a factor of $P(x)$?

ii) Is $x-2$ a factor of $P(x)$?

Example 2

When $2x^3 + mx^2 + nx - 6$ is divided by $x - 2$, the remainder is 20. The same polynomial has a factor of $x + 2$. Determine the values of m and n.

Possible Factors Of A Polynomial

- When factoring a polynomial $P(x)$, it is helpful to know which integer values of a to try when determining if $P(a)=0$.

Integral Zero Theorem

- If $x-a$ is a factor of a polynomial function with integral coefficients, then it is a factor of the constant term of $P(x)$.

Example 3

(i) Given $P(x)=x^3-4x^2+x+6$ List the possible integer values for the zeros of the function.

(ii) Apply the factor theorem to determine one factor

(iii) Use synthetic division to determine the remaining factors.

Example 4

Factor the following polynomials:

i) $P(x) = 2x^3 - 5x^2 - 4x + 3$

ii) $P(x) = x^4 + 4x^3 - 7x^2 - 34x - 24$

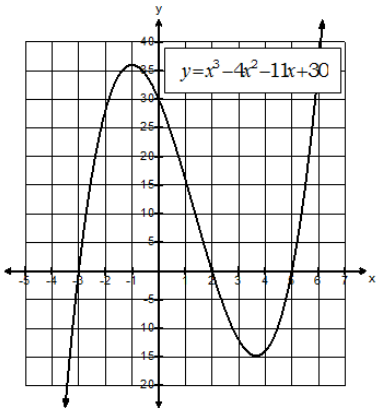
(iii) $P(x) = 2x^3 - 6x^2 + 8$

Your Turn

Factor the polynomial $x^4 - 5x^3 + 2x^2 + 20x - 24$

Your Turn

What is the factored form of $x^3 - 4x^2 - 11x + 30$? How can you use the graph of the corresponding polynomial function to simplify your search for integral roots?



Textbook Questions pg.133-135 #1ab, 2ae, 4ae, 5bd, 6ae, 7b, 11, 13, 15

Other Techniques to Factor Higher Order Polynomials

1. greatest common factor
2. similarity between quadratics and quartics
3. grouping

Example 1

Factor the following polynomials:

i) $y = 2x^3 - x^2 - 6x$

ii) $y = x^4 - 5x^2 + 4$

iii) $y = 3x^5 - 15x^3 + 12x$

iv) $y = x^3 - 2x^2 - 9x + 18$

- even # of terms
- group the four terms to form two pairs where they have the same ratio