Lesson 3.2: The Remainder Theorem

Review: Factoring and solving polynomial functions

↳ What strategies did you use in Math 2200 to factor quadratics?

Example 1

Solve the following polynomial functions:

$i)\ 0 = x^2 - 7x - 18$

$ii)\ 0 = 2x^2 - 5x - 3$

$iii)\ 0 = 12x^3 - 2x^2 - 4x$

Develop strategies to factor cubic, quartic and quintic functions

- » long division
- » synthetic division

Example 2

(i) Review long division with numerical expressions $327 \div 12$

(ii) Identify the divisor, dividend, quotient and remainder.

$$\text{divisor}\overline{)\begin{array}{r}\text{quotient}\\ \text{dividend}\\ \hline \text{remainder}\end{array}}$$

(iii) Check: dividend =(divisor x quotient) + remainder

Think About:

$$\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}$$

Example 3

(i) Perform long division $180 \div 12$

(ii) What is the value of the remainder? What does this value mean?

You can divide polynomials by other polynomials using the same long division process that use you to divide numbers.

Example 4

(i) Divide the polynomial $x^2 + x - 6$ by $x + 3$.

(ii) Express the division in the form $P(x) = Q(x) \times D(x) + R$ to verify your answer.

Think About: $\frac{polynomial}{divisor} = quotient + \frac{remainder}{divisor}$

Divide a polynomial by a binomial of the form $x-a$

Example 5

(i) Divide the polynomial $P(x)=x^3-4x^2+x+6$ by $x+1$

factor:
root:

Long Division

Synthetic Division

- descending order
- coefficients of the dividend
- substitute 0 for any missing terms

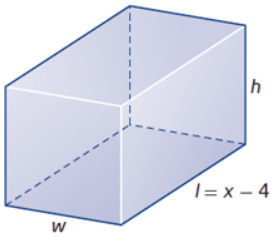
(ii) What is the value of the remainder? Write the polynomial in factored form.

Example 6

Use synthetic division to determine the quotient when $P(x)=x^3-28x-48$ is divided by $x+4$

Example 7

The volume of a rectangular prism is given by $V(x) = x^3 + 3x^2 - 36x + 32$. Determine the possible measures for w and h in terms of x if the length (l) is represented by $x - 4$.



Your Turn

(a) Divide $P(x) = x^3 - 2x^2 - 5x + 6$ by $x - 3$.

Express the equation in the form $P(x) = (x - a) \cdot Q(x) + R$

(b) Divide $P(x) = x^3 + 2x^2 - 5x + 4$ by $x - 2$.

Express the equation in the form $\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}$

Remainder Theorem

↳When a polynomial $P(x)$ is divided by a binomial $x-a$ the remainder is $P(a)$.

Think About:

Polynomial = Divisor x Quotient + Remainder

$$P(x)=(x-a)Q(x)+R$$

Determine $P(a)$: ⟶

Example 8

(i) Divide x^3+5x^2+8x+4 by $x+2$ and state the remainder.

(ii) Substitute the value $x=-2$ into the polynomial x^3+5x^2+8x+4 and evaluate the result.

Your Turn

Find the remainder when $P(x)=2x^{100}-3x^2+5$ is divided by $(x+1)$?

⟶

Example 9

Find the value of k when $P(x) = kx^3 - x - 3$ is divided by $(x-1)$, the remainder is -1.

Your Turn

Find the value of k when $(2x^3 + 3x^2 + kx - 3)$ is divided by $(2x+5)$, the remainder is 2.

Example 10 p.125 #15

When the polynomial $3x^3 + ax^2 + bx - 9$ is divided by $x - 2$, the remainder is -5. When it is divided by $x + 1$, the remainder is -16. What are the values of a and b?

TextBook Questions:

pg. 124-125 #4ac, 5a, 6ac, 7ad, 8ac, 9, 10, 14