CHAPTER 3

Polynomial Functions

Lesson 3.1: Characteristics of Polynomial Functions

Defining a Polynomial Function

Constant Function → $f(x) = 4$

Linear Function → $f(x) = 3x + 1$

Quadratic Function → $f(x) = 2x^2 + 4x - 1$

↳ Look at polynomials with a degree that is greater than 2

Cubic Polynomial → $f(x) = -5x^3 + 4x^2 + 2x - 1$

Quartic Polynomial → $f(x) = 2x^4 + 4x - 1$

Quintic Polynomial → $f(x) = 2x^5 - 3x^4 - x^3 + 4x^2 + 6x + 2$

Note: The polynomial function is written in descending order.

Polynomial Function

↳ A polynomial function is a function that can be written in the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + ...a_2 x^2 + a_1 x + a_0$

Example: $P(x) = 4x^3 + 6x^2 - 5x + 7$

The parts of the function are defined as follows:

- the values of $a_1, a_2, ...a_n$ are the ____________
- the term a_0 is the _________ of the graph and is referred to as the ________
- the coefficient of the highest power of x is called the ________________
- the exponents are ________________
- the largest exponent is the __________ of the polynomial function

Note:

Expressions containing roots of variables, negative powers, fractional powers, or any powers other than whole numbers are not polynomial functions.

Your Turn

Identify whether each function is a polynomial function.

$(a)\ y = \frac{1}{x}$

$(b)\ y = -4x^5 + 2x^3 + 1$

$(c)\ y = \sqrt{x} + x^2 - 2$

$(d)\ y = 5^x + 2$

Classifying Polynomial Functions

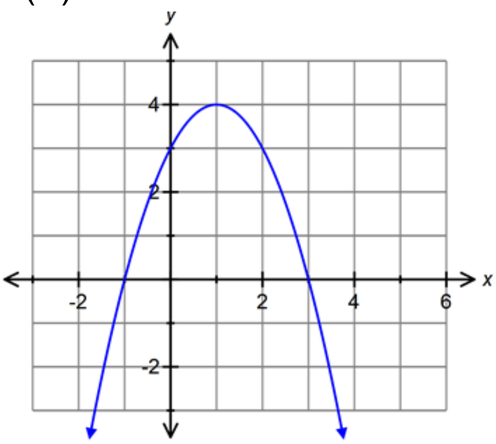
↳ Polynomial functions and their graphs can be classified by

- » identifying the type of function
- » number of turns
- » the degree
- » number of x-intercepts
- » the end behaviour

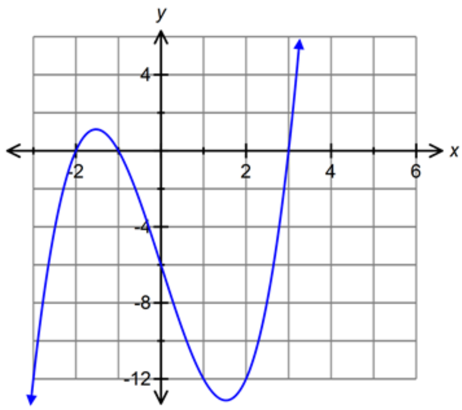
End Behaviour

⟶ refers to the behaviour of the *y*-values of the function as *x* becomes very large or very small $(x \to \pm\infty)$

(A)



(B)



Absolute and Relative Maximum/Minimum Value

⟶ Absolute max/min: the highest or lowest point over the entire domain of the function.

Graph A:

Graph B:

⟶ Relative max/min: the highest or lowest point in a particular section of the graph.

Graph A:

Graph B:

Even Degree Polynomial Functions

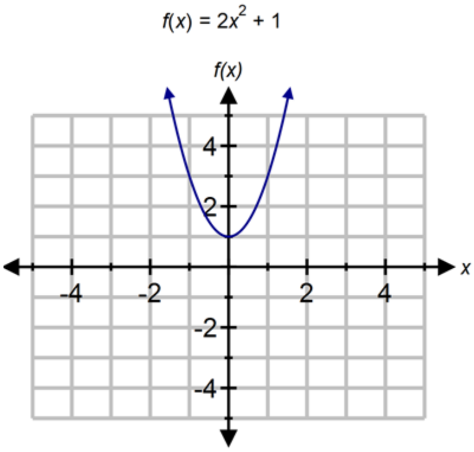
(a) Generalize the end behaviour of the function

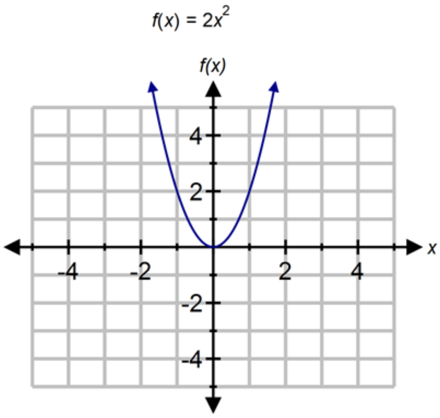
(b) Possible number of x-intercepts

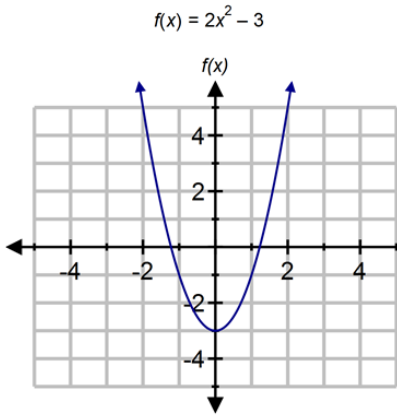
(c) Identify the number of turning points

(d) Relationship between the y-intercept of the graph and the constant term in the equation.

Degree 2: Quadratic Function





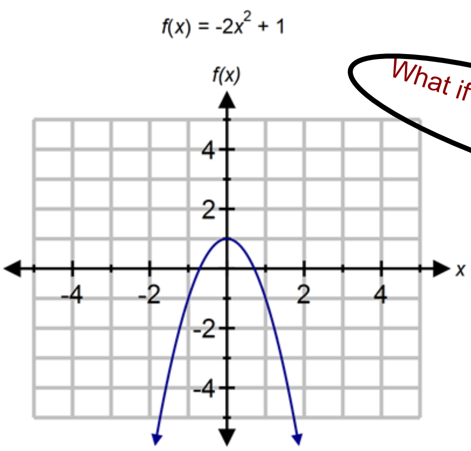


possible # of x-int:__________

# of turning points:__________

y-int:_______

End Behaviour

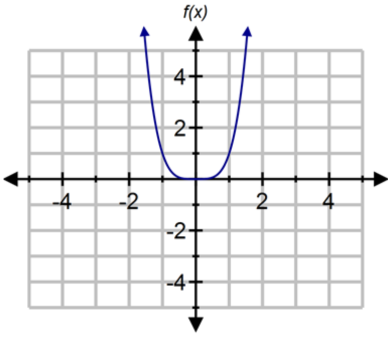


What if the leading coefficient was negative?

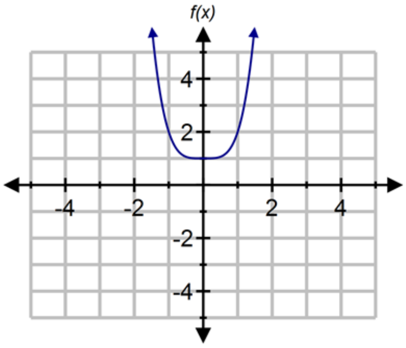
End Behaviour

Degree 4: Quartic Function

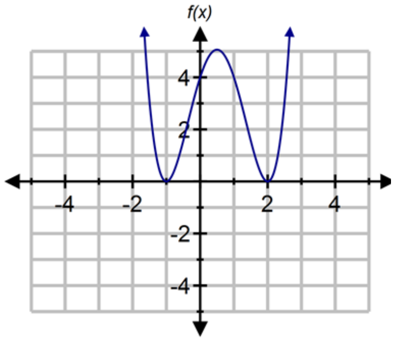
$f(x) = x^4$



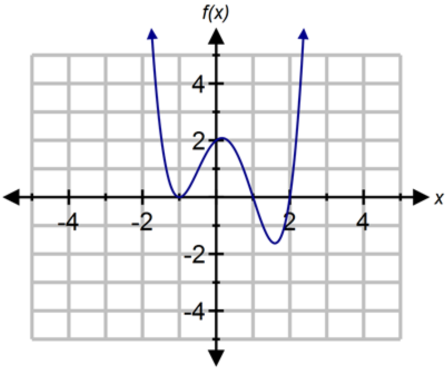
$f(x) = x^4 + 1$



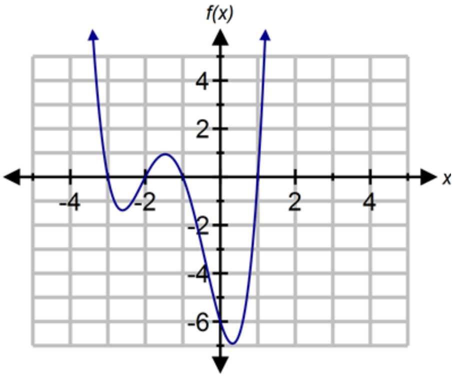
$f(x) = (x-2)^2(x+1)^2$



$f(x) = (x+1)^2(x-1)(x-2)$



$f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$



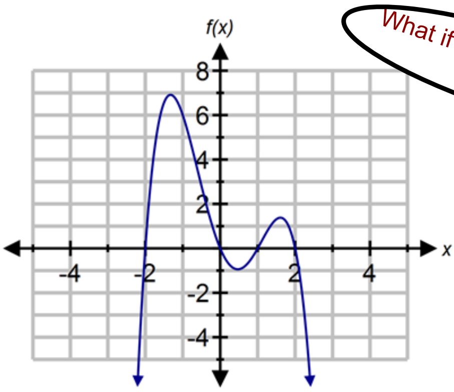
possible # of x-int:_________

# of turning points:_________

y-int:______

End Behaviour

$f(x) = -x^4 + x^3 + 4x^2 - 4x$

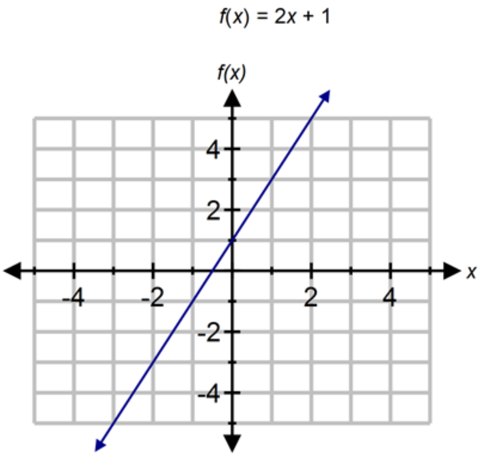


What if the leading coefficient was negative?

End Behaviour

Odd Degree Polynomial Functions

Degree 1: Linear Function

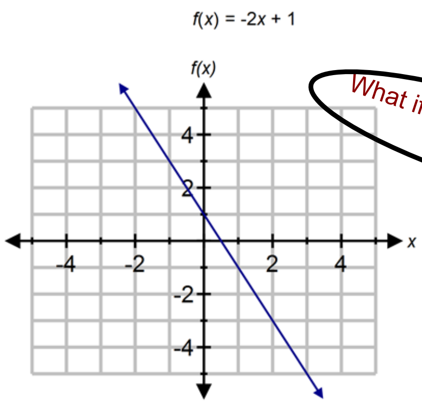


possible # of x-int:__________

# of turning points:__________

y-int:_______

End Behaviour

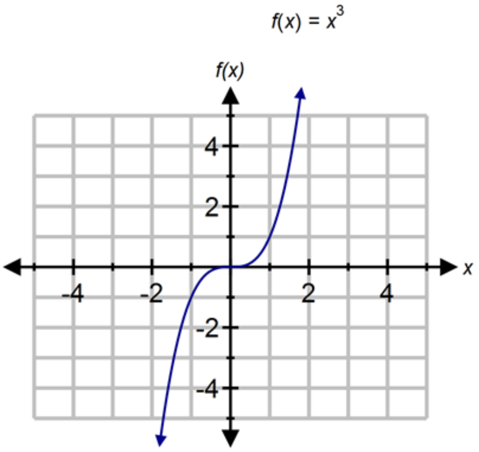


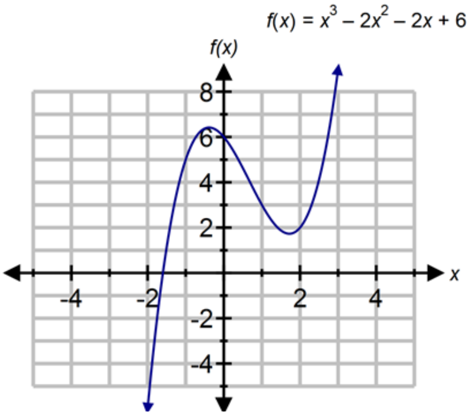
What if the leading coefficient was negative?

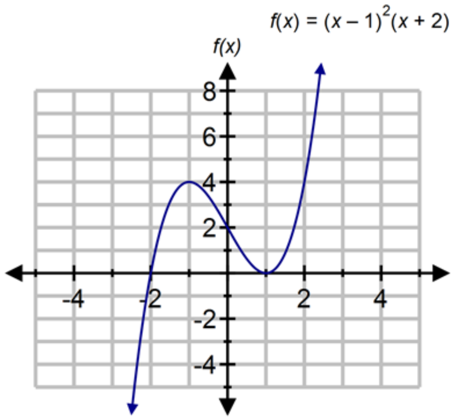
End Behaviour

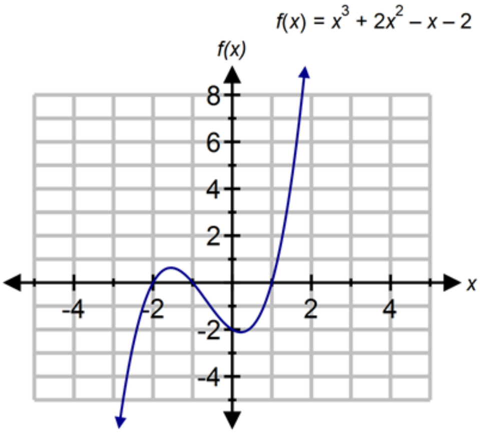
→

Degree 3: Cubic Function







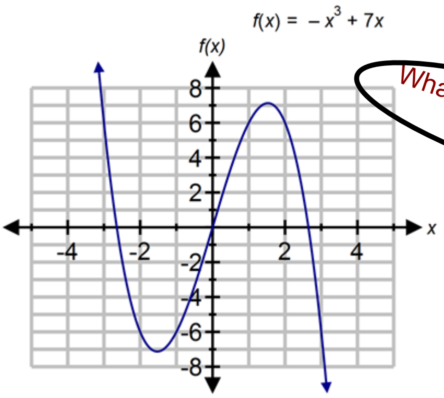


possible # of x-int:__________

# of turning points:__________

y-int:______

End Behaviour

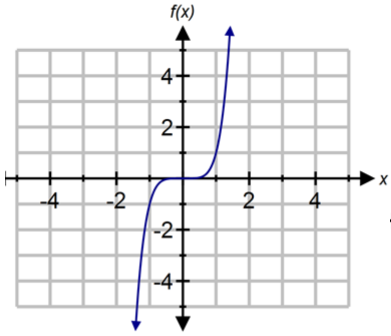


What if the leading coefficient was negative?

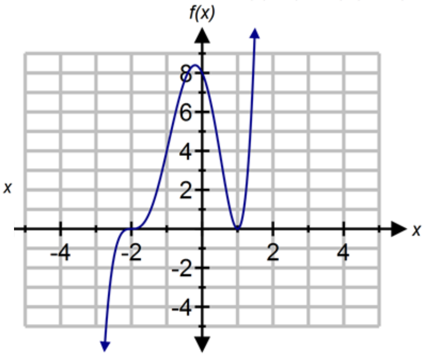
End Behaviour

Degree 5: Quintic Function

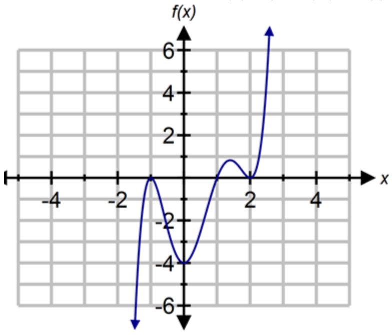
$f(x) = x^5$



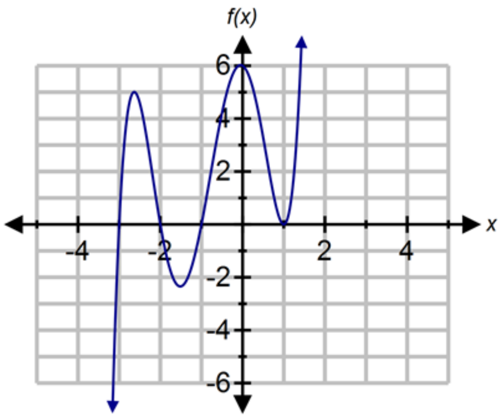
$f(x) = (x + 2)^3(x - 1)^2$



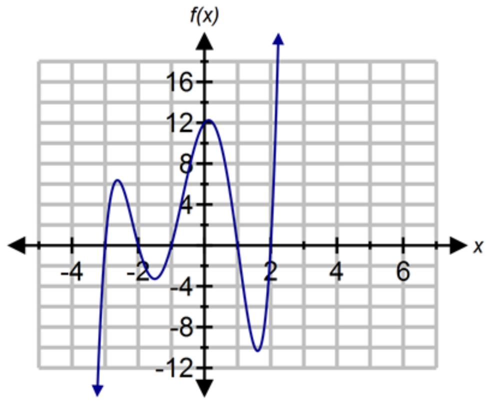
$f(x) = (x + 1)^2(x - 1)(x - 2)^2$



$f(x) = (x + 3)(x + 2)(x + 1)(x - 1)^2$



$f(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$



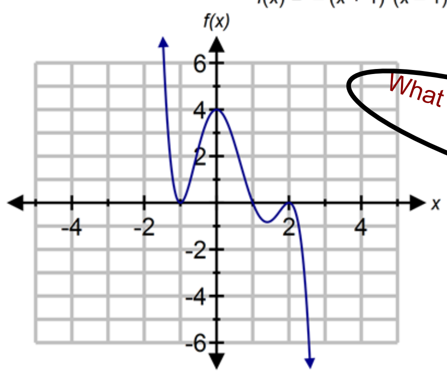
possible # of x-int:__________

# of turning points:__________

y-int:_______

End Behaviour

$f(x) = -(x + 1)^2(x - 1)(x - 2)^2$



What if the leading coefficient was negative?

End Behaviour

Summary

Graphs of Polynomials

Type of Polynomial	Degree (n)	Number of x-intercepts	Number of turning points (n-1)
linear	1	1	0
quadratic	2	from 0 to 2	1
cubic	3	from 1 to 3	0 or 2
quartic	4	from 0 to 4	1 or 3
quintic	5	from 1 to 5	0, 2 or 4

- y-intercept of the graph is the constant term of the polynomial function
- end behaviour depends on whether the leading coefficient is positive or negative
- odd functions must have at least one x-intercept

odd functions:

If the leading coefficient is positive graph extends from Q3 to Q1
If the leading coefficient is negative graph extends from Q2 to Q4

opposite left & right

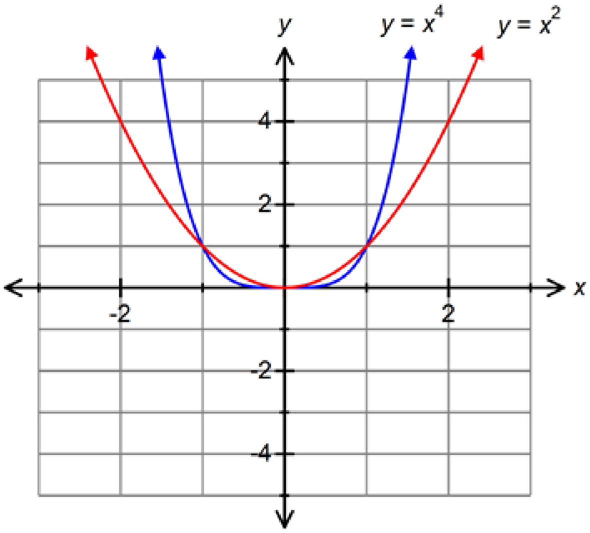
even functions:

If the leading coefficient is positive graph extends from Q2 to Q1

If the leading coefficient is negative graph extends from Q3 to Q4

same left & right

Compare $y = x^2$ and $y = x^4$



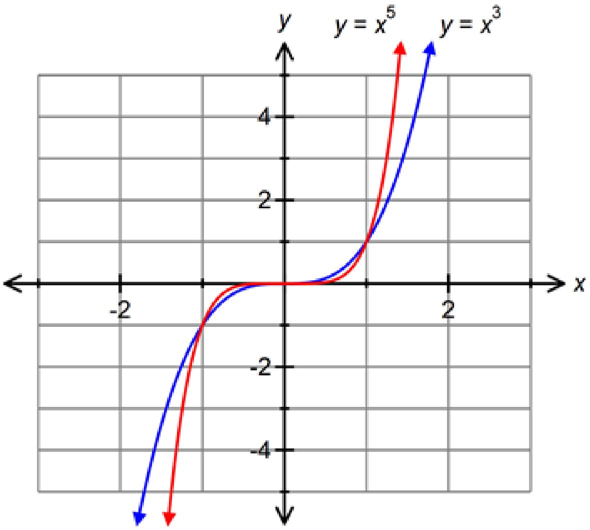
Simplest graphs are the monomial functions $f(x) = ax^n$

n ⟶ even   similar to $y = x^2$

n ⟶ odd   similar to $y = x^3$

The greater the value of n, the <u>flatter</u> the graph of the monomial is on the interval $-1 \le x \le 1$

Compare $y = x^3$ and $y = x^5$



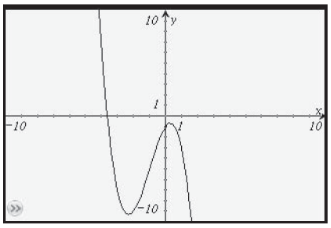
no turning point

Your Turn

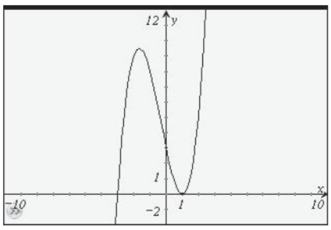
Identify the following characteristics of the graph for each polynomial function:

Function	Type of Function	Odd or Even?	Leading Coefficient	End Behaviour	# of possible x-intercepts	y-int	Matching Graph
$a. P(x) = 2x^3 - x^2 + 5x + 3$	cubic	odd	2	Quad III to Quad I	at least 1 x-int and at most 3 x-int (1, 2 or 3)	y = 3	B
$b. P(x) = -x^4 + x^2 - 2x - 5$							
$c. P(x) = 4x^2 - 2x^3 - 2x - 1$							
$d. P(x) = -3x^5 + x^2 - 4x$							

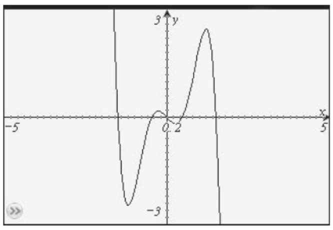
A



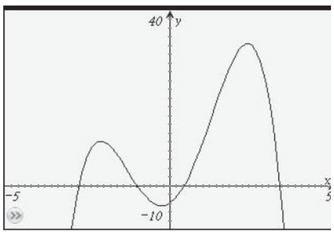
B



C



D



TextBook Questions: pg. 114-115 #1, 2, 3abc, 4abc, 5