Lesson 1.4: Inverse of a Relation

Inverse of a relation ⟶ is a rule for reversing what the function does (reverse the operation and the order)

Examples:

Notation: f^{-1}

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$

superscript in this situation does not mean raising to a power

(i) $f(x) = x + 2$ $\quad$ $f^{-1}(x) = x - 2$

(ii) $f(x) = 5x$ $\quad$ $f^{-1}(x) = \frac{x}{5}$

(iii) $f(x) = 2x + 3$ $\quad$ $f^{-1}(x) = \frac{x-3}{2}$

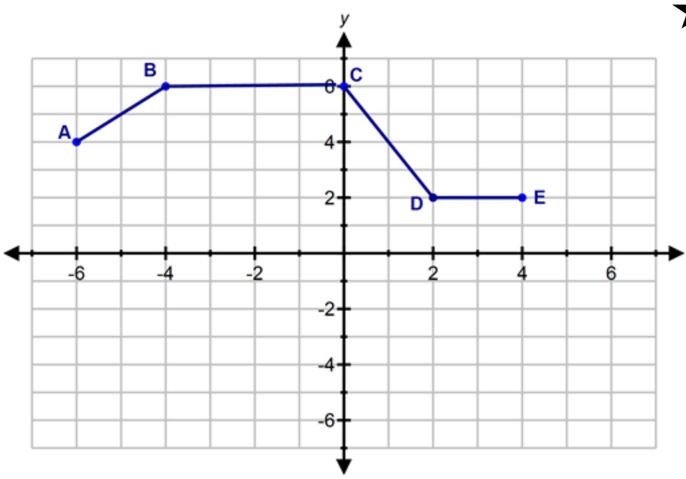
(iv) $f(x) = x^2$ $\quad$ $f^{-1}(x) = \sqrt{x}, x \geq 0$

Example 1

(i) Sketch the graph of its inverse relation.

To graph the inverse relation ⟶ <u>interchange</u> the x and y coordinates of key points on the graph of the relation

Points on the relation	Points on the inverse relation
(-6,4)	
(-4,6)	
(0,6)	
★ (2,2)	
(4,2)	



(ii) State the domain and range of the relation and its inverse.

	Domain	Range
Relation		
Inverse Relation		

(iii) Determine whether the relation and its inverse are functions.

Vertical Line Test ⟶ there is only one value of y for each value of x

Horizontal Line Test

Example 2

(i) Sketch the graph of its inverse relation.

(ii) State the domain and range of the relation and its inverse.

(iii) Determine whether the relation and its inverse are functions.

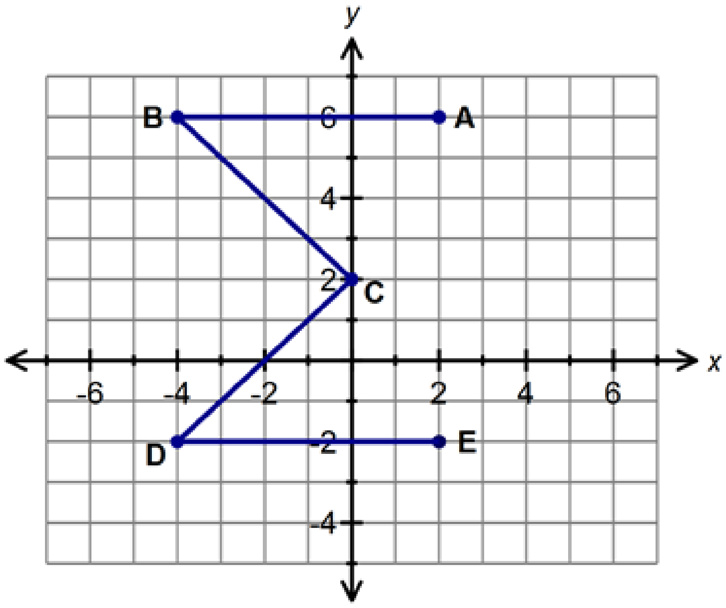
$A(2,6) \rightarrow A'(\qquad)$

$B(-4,6) \rightarrow B'(\qquad)$

$C(0,2) \rightarrow C'(\qquad)$

$D(-4,-2) \rightarrow D'(\qquad)$

$E(2,-2) \rightarrow E'(\qquad)$



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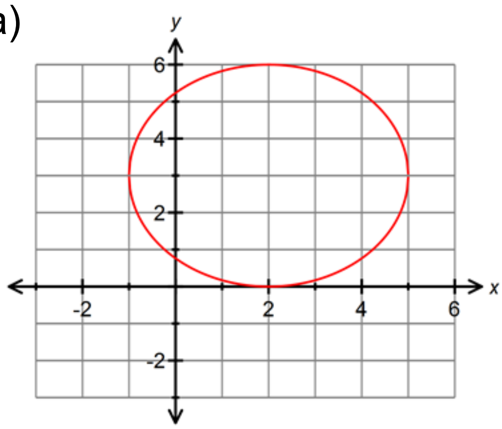
Example 3

State whether or not the graph of the relation is a function. Without graphing the inverse of the relation, determine whether the inverse of the relation is a function.

(a)



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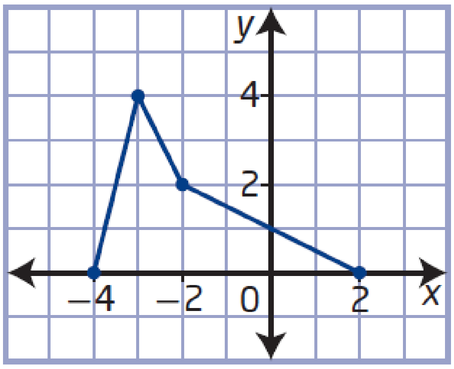

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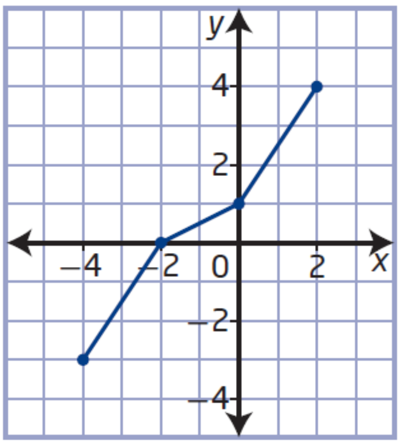
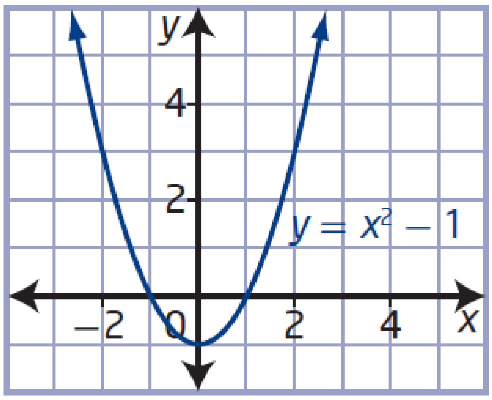


(d)

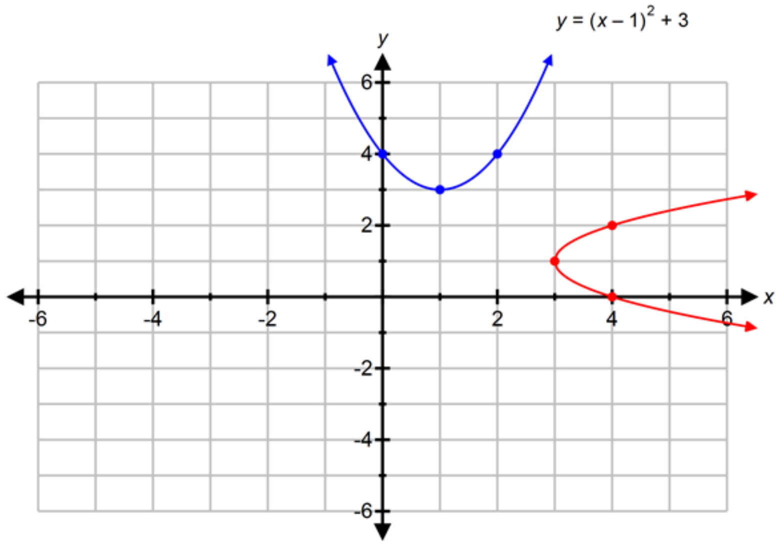


Assign P.52 #3

Restricting The Domain

Example 4

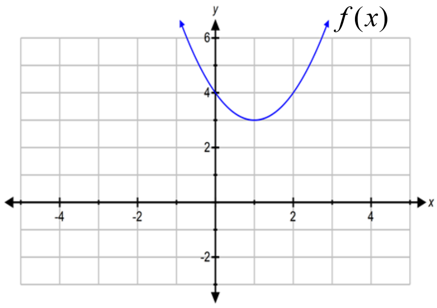
Consider the function $y=(x-1)^2+3$ and its inverse.



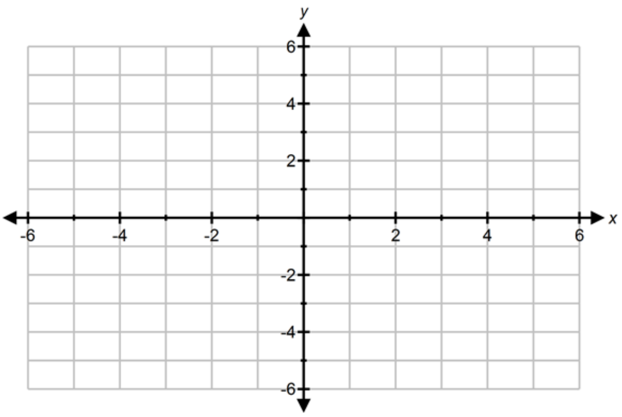
(i) Algebraically determine the equation of the inverse.

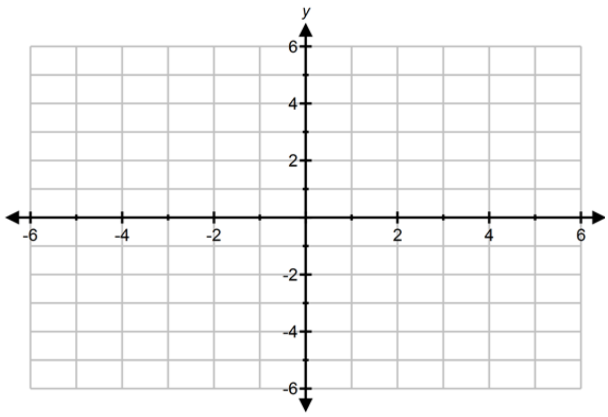
(ii) Is the inverse of $f(x)$ a function.

(iii) Describe how the domain of $f(x)$ could be restricted so that the inverse of $f(x)$ is a function.

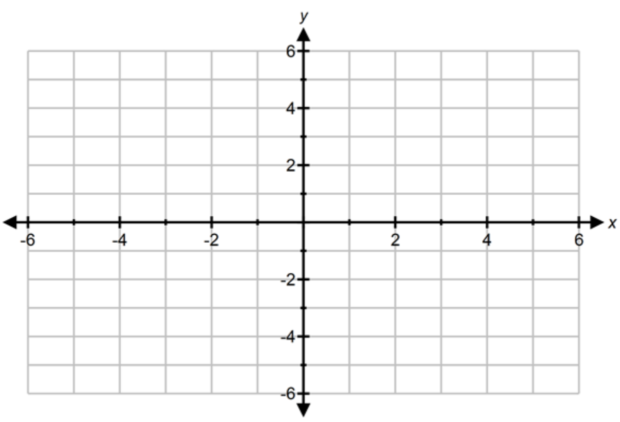


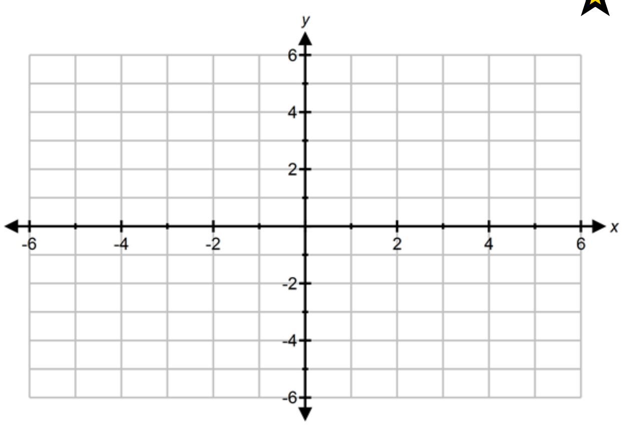
Restrict Domain of Original Function





Restrict Domain of Original Function





Example 5

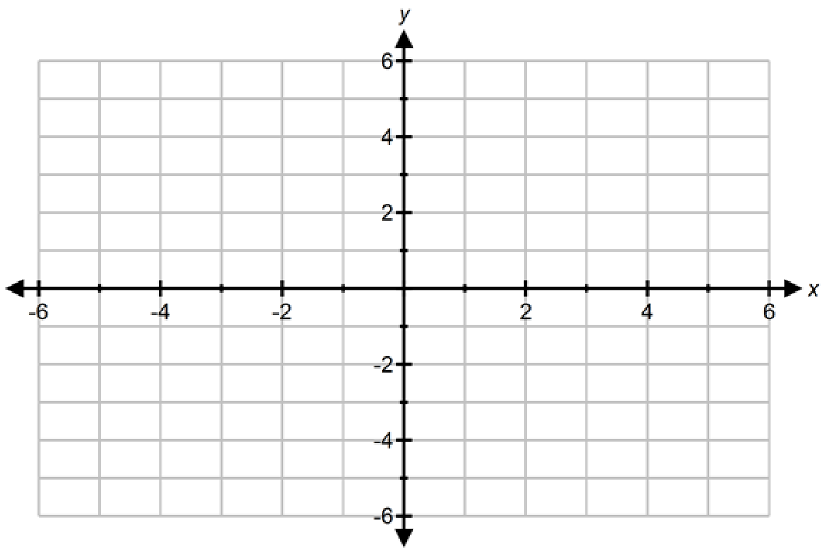
Determine algebraically the inverse of the function $f(x) = 3x^2 - 6x + 1$. Restrict the domain so that the inverse of $f(x)$ is also a function. Verify by sketching the graph of the restricted function and its inverse.

Equation

Restricted Domain

Inverse Function

Graph



Assign P.52-54 #4ac, 11, 12aef, 14bcd

Example 6

For each of the following, determine the equation of the inverse of $f(x)$

(a) $f(x) = 3x - 2$

(b) $f(x) = x^2 - 4$

(c) $f(x) = (x-2)^2 + 1$ (vertex form)

Example 6 cont'd

(d) $f(x) = x^2 - 8x + 11$ (Goal: rewrite in vertex form)

(e) $y = 2x^2 - 8x + 11$

<u>Question</u>

How would your answers change if the question asked:

What is the equation of the inverse $f^{-1}(x)$?

Example 7

Given $f(x) = (x-3)^2 + 2$ determine the value of $f^{-1}(4)$.

Assign P.52-54 #5, 6, 15ab