

### *Lesson 1.3: Combination of Transformations*

$$y - k = af(b(x - h))$$

Multiple transformations can be applied to the same function.  
The **order** of transformations:

- « multiplication/division (stretches/reflections)
- « addition/subtraction (translations)

The following four step process may help you keep organized:

**Step 1:** horizontal stretch by a factor of  $\left|\frac{1}{b}\right|$

**Step 2:** vertical stretch by a factor of  $|a|$

**Step 3:** reflection in the  $y$ -axis (if  $b < 0$ )  
reflection in the  $x$ -axis (if  $a < 0$ )

**Step 4:** horizontal and/or vertical translation ( $h$  and  $k$ )

$$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

## Lesson 1.3 Combination of Functions

### Example 1

Describe the transformations that must be applied to the function  $y = f(x)$  to obtain the transformed function  $y = 2f(-3(x+1)) - 2$ . Sketch the graph.

Identify the parameters:

$$a =$$

$$b =$$

$$VS =$$

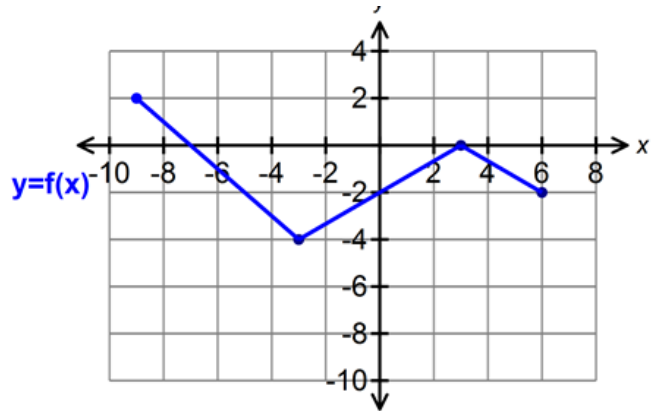
$$HS =$$

$$k =$$

$$h =$$

$$VT =$$

$$HT =$$



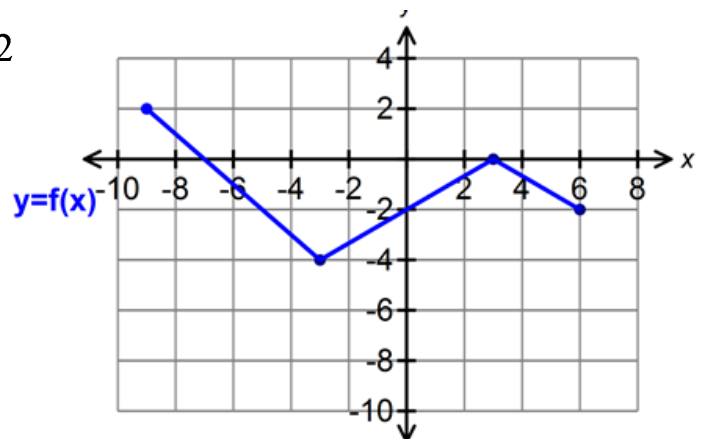
reflection?

Mapping Rule:  $(x, y) \rightarrow$

| $y = f(x)$ |   |
|------------|---|
| x          | y |
|            |   |
|            |   |
|            |   |
|            |   |

 $\rightarrow$ 

| $y = 2f(-3(x+1)) - 2$ |   |
|-----------------------|---|
| x                     | y |
|                       |   |
|                       |   |
|                       |   |
|                       |   |



Using Transformations:

$(-9, 2) \rightarrow$

## Lesson 1.3 Combination of Functions

### Example 2

Sketch the graph of the transformed function  $y = \frac{1}{2}f\left(-\frac{1}{2}(x + 2)\right) - 1$

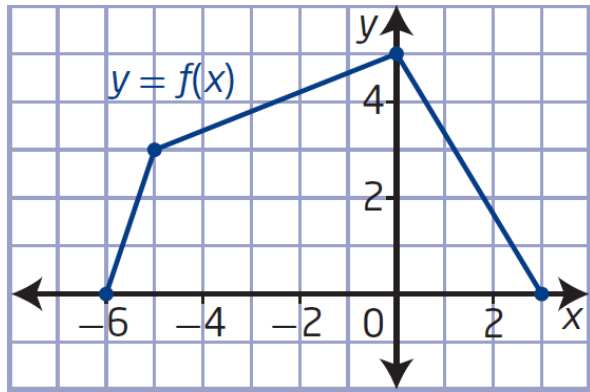
$a =$

$b =$

reflection:

$h =$

$k =$



$(x,y) \longrightarrow$

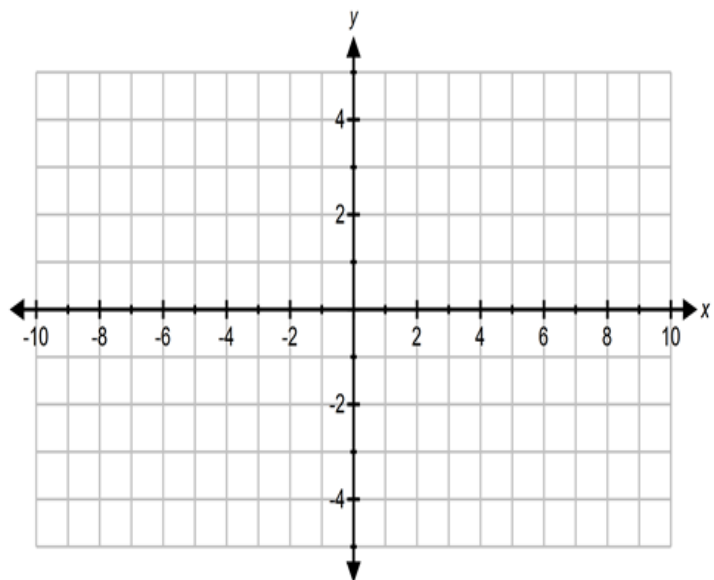
### Transformed Points

A(-6,0)  $\longrightarrow$

B(-5,3)  $\longrightarrow$

C(0,5)  $\longrightarrow$

D(3,0)  $\longrightarrow$



Example 3

Identify the transformations and write the mapping rule for the following:

(i)  $y - 6 = 3f(4x - 8)$

(ii)  $y = f(-x - 8)$

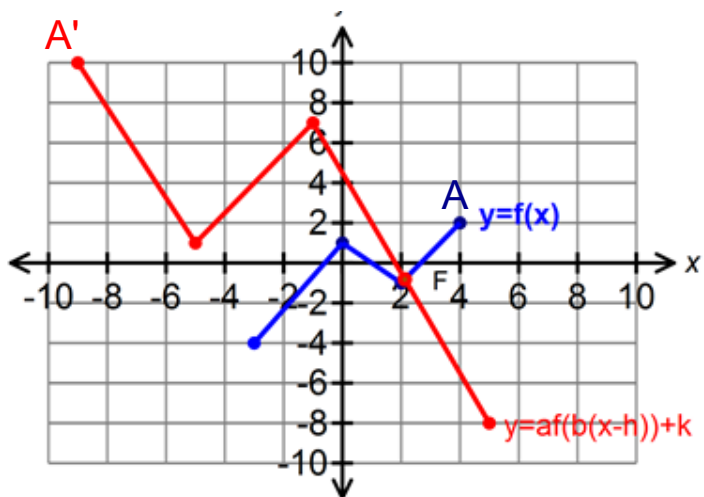
(iii)  $-3y - 6 = f(2(x + 1))$

(iv)  $y = f(4(2x - 1))$

Example 4

Determine the equation for the image of  $y = f(x)$  in the form

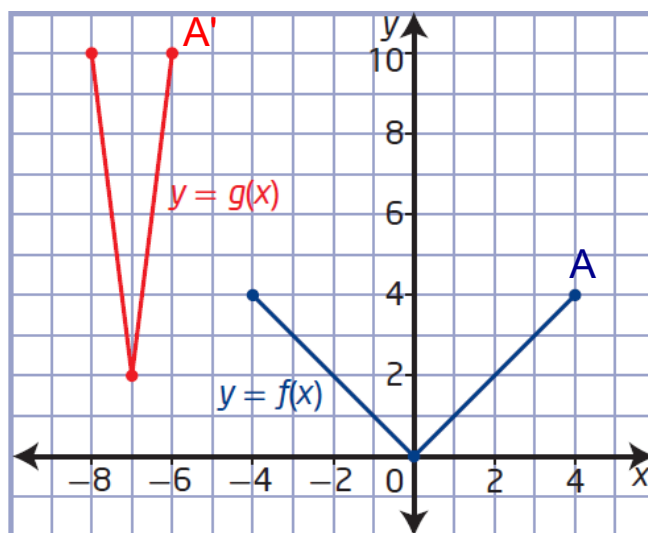
$$y = af(b(x-h)) + k$$



## Your Turn

The graph of the function  $y = g(x)$  represents a transformation of the graph of  $y = f(x)$ . Determine the equation of  $g(x)$  in the form

$$y = af(b(x - h)) + k$$



Your Turn 

Describe the transformations that can be applied to  $f(x)$  to obtain the graph of the transformed function and write the mapping rule.



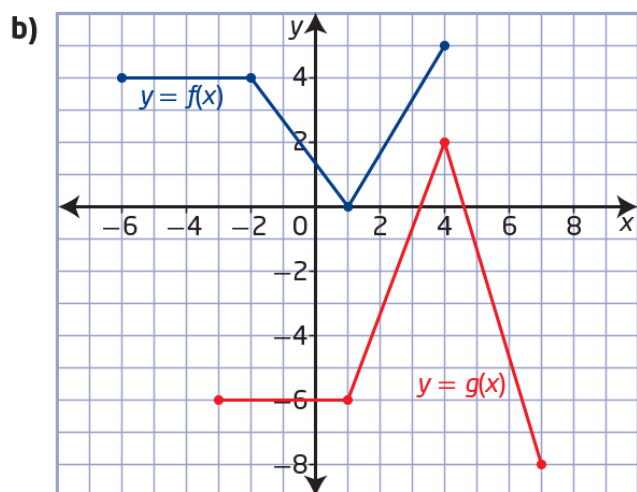
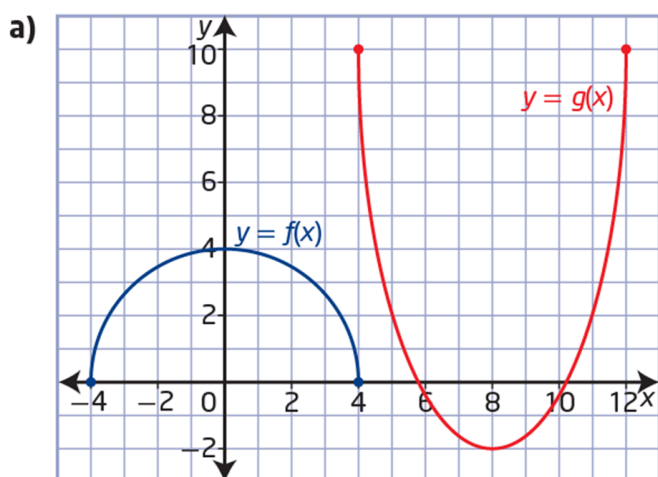
| Equation                                   |                 | Mapping Rule  |
|--|-----------------|---|
| (a) $y - 6 = f(3x)$                        | $y = f(3x) + 6$ | $(x, y) \rightarrow \left(\frac{1}{3}x, y + 6\right)$ |
| (b) $4y = f\left(-\frac{1}{2}(x-1)\right)$ |                 |   |
| (c) $y = 3f(-x-2)$                         |                 |   |
| (d) $-\frac{1}{2}y = f(x+1)$               |                 |   |
| (e) $y = 3f(4x-8) + 2$                     |                 |   |
| (f) $2y - 8 = f(-3(x-2))$                  |                 |   |
| (g) $2y - 5 = f(3x-2)$                     |                 |   |
| (h) $-\frac{1}{3}y + 1 = f(x+4)$           |                 |   |

## Lesson 1.3 Combination of Functions

Assign p.39-41 #4a, 6cde, 7acf, 8ab, 10abc, 12ab

P.40 #10

$g(x)$  is the transformed function





Lesson 1.3 Combination of Functions

