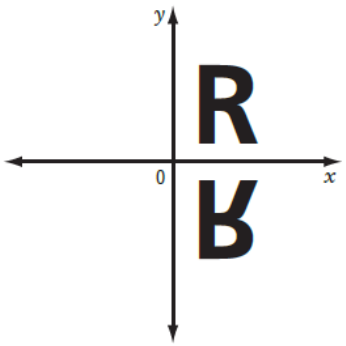
Lesson 1.2: Reflections and Stretches

↳ A reflection creates a mirror image of the graph of a function across a line of reflection.

1. Horizontal Reflection:



$y = -f(x)$

or

$-y = f(x)$

« output is multiplied by -1

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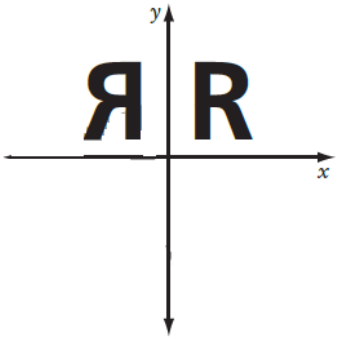
2. Vertical Reflection:



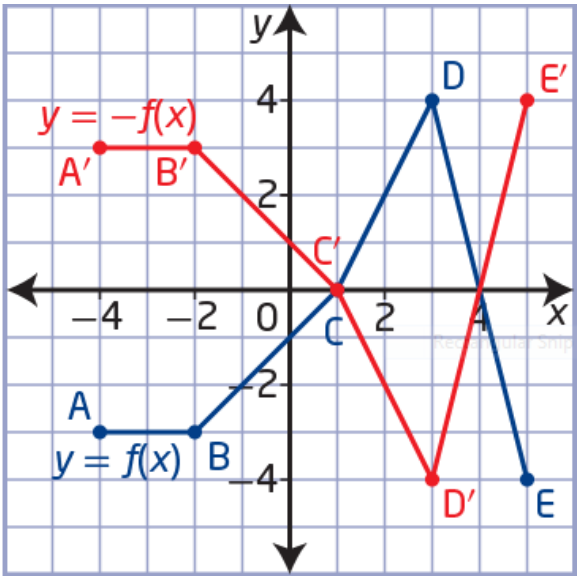
$y = f(-x)$

« input is multiplied by -1

→

Reflection on x-axis

Example 1



A (-4, -3) ⟶ A'

B (-2, -3) ⟶ B'

★C (1, 0) ⟶ C'

D (3, 4) ⟶ D'

E (5, -4) ⟶ E'

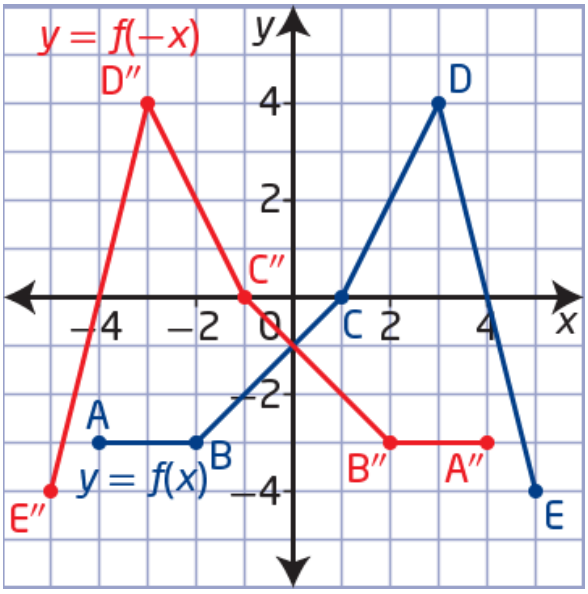
(i) How do the coordinates of the point change?

(ii) Write the mapping rule.

(iii) Write the equation of the reflected graph compared to the graph $y = f(x)$.

Reflection on y-axis

Example 2



A (-4, -3) ⟶ A"

B (-2, -3) ⟶ B"

C (1, 0) ⟶ C"

D (3, 4) ⟶ D"

E (5, -4) ⟶ E"

(i) How do the coordinates of the point change?

(ii) Write the mapping rule.

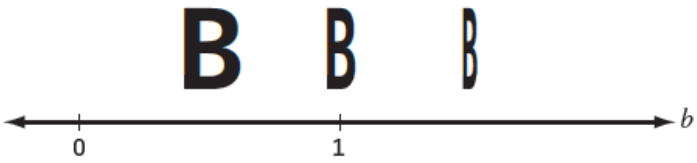
(iii) Write the equation of the reflected graph compared to the graph $y = f(x)$.

Stretches

affect x-value

1. Horizontal Stretch $y = f(bx)$

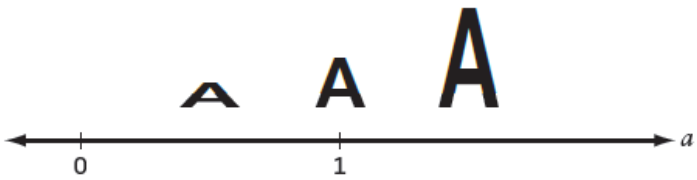
↳ the function is narrower (compression) or wider (expansion)



affect y-value

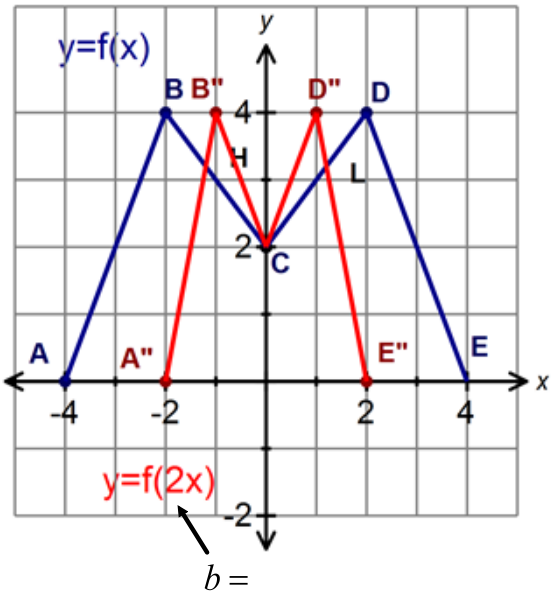
2. ***Vertical Stretch*** $y = af(x)$

↳ the function is shorter (compression) or taller (expansion)



Horizontal Stretch: $y = f(bx)$

Example 3



A (-4, 0) → A" (-2,0)

B (-2, 4) → B" (-1,4)

★ C (0, 2) → C" (0,2)

D (2, 4) → D" (1,4)

E (4, 0) → E" (2,0)

(i) How do the coordinates of the point change?

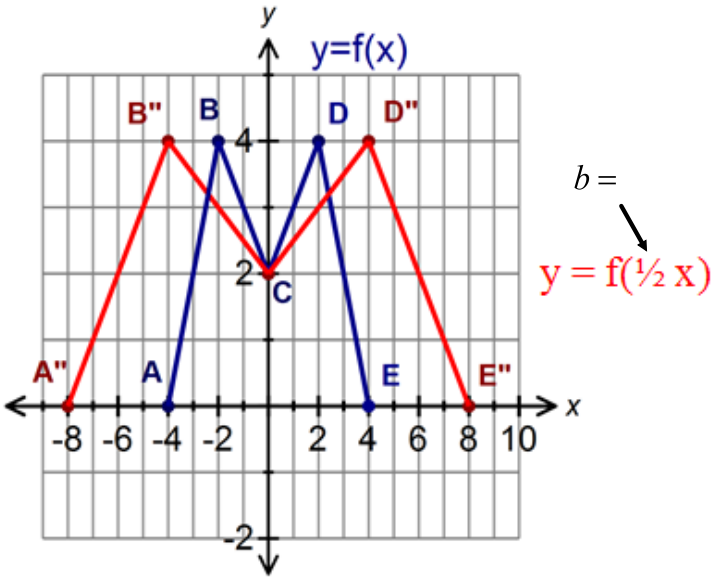
(ii) What is the horizontal stretch factor?

(iii) Write the mapping rule.

(iv) Write the equation of the graph compared to the graph $y = f(x)$.
Compare the domain of each function.

(v) Why do the y-intercept map to itself?

Example 4



A (-4, 0) → A" (-8,0)

B (-2, 4) → B" (-4,4)

C (0, 2) → C" (0,2)

D (2, 4) → D" (4,4)

E (4, 0) → E" (8,0)

(i) How do the coordinates of the point change?

(ii) What is the horizontal stretch factor?

(iii) Write the mapping rule.

(iv)Write the equation of the graph compared to the graph $y = f(x)$.
Write the domain of each function.

Summary:

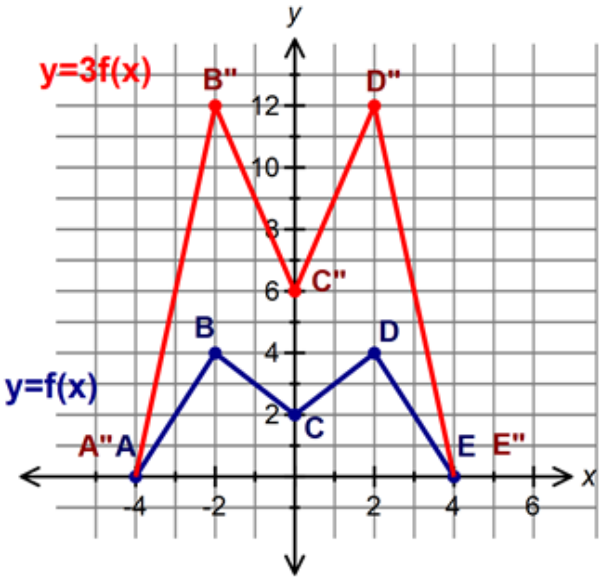
$y = f(bx)$ Function

$\frac{1}{|b|}$ Horizontal Stretch Factor

Mapping Notation: $(x, y) \to (\frac{1}{b}x, y)$

Vertical Stretch

Example 5



A (-4, 0) → A" (-4,0)

B (-2, 4) → B" (-2,12)

C (0, 2) → C" (0,6)

D (2, 4) → D" (2,12)

E (4, 0) → E" (4,0)

(i) How do the coordinates of the point change?

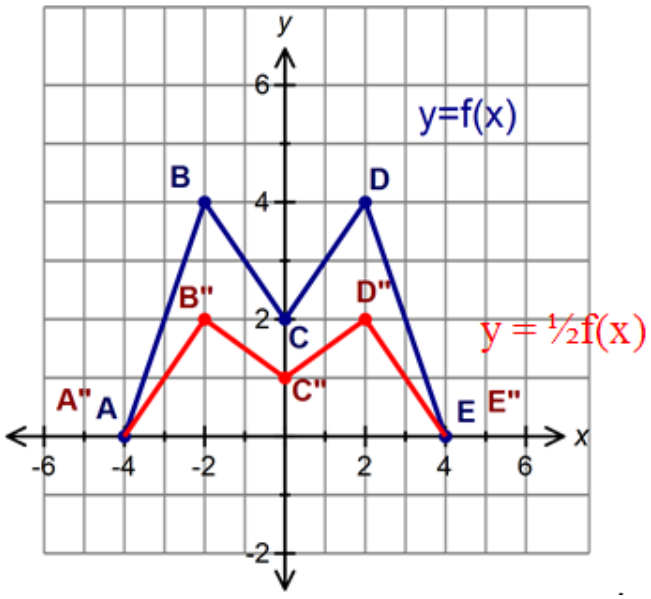
(ii) What is the vertical stretch factor?

(iii) Write the mapping rule.

(iv) Why do the x-intercept map to itself?

(v) Write the equation of the graph compared to the graph $y = f(x)$.
Compare the range of each function.

Example 6



A (-4, 0) → A" (-4,0)

B (-2, 4) → B" (-2,2)

C (0, 2) → C" (0,1)

D (2, 4) → D" (2,2)

E (4, 0) → E" (4,0)

(i) How do the coordinates of the point change?

(ii) What is the vertical stretch factor?

(iii) Write the mapping rule.

(iv) Write the equation of the graph compared to the graph $y = f(x)$.
Compare the range of each function.

Summary:

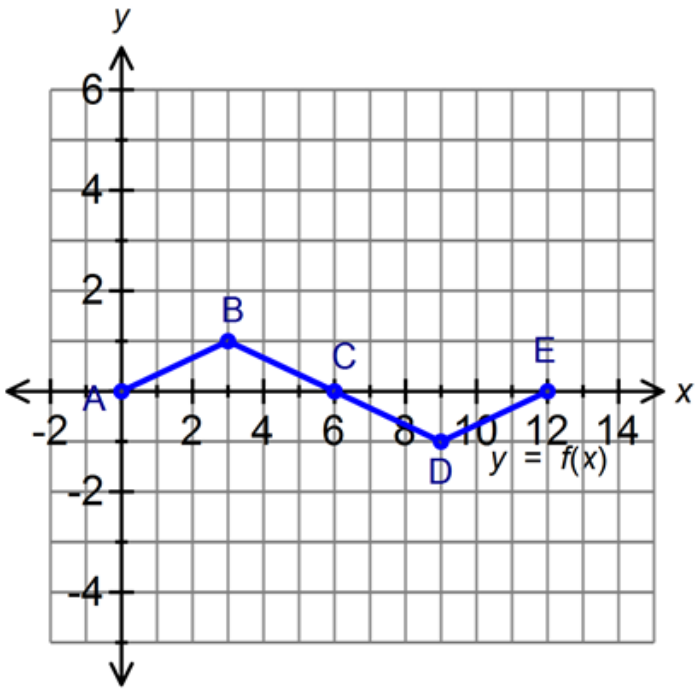
$y = af(x)$ Function

$|a|$ Vertical Stretch Factor

Mapping Notation: $(x, y) \to (x, ay)$

Example 7

Given the graphs $y = f(x)$ and $y = 5f(3x)$ on the same set of axes.
Write the mapping notation representing the transformation.



$y = 5f(3x)$

$a =$ _____ represents a vertical stretch by a factor of _____

$b =$ _____ represents a horizontal stretch by a factor of _____

(x,y) ⟶

Apply the vertical stretch followed by the horizontal stretch.
to determine the key points of the transformed graph.

A(0,0) ⟶ A'

B(3,1) ⟶ B'

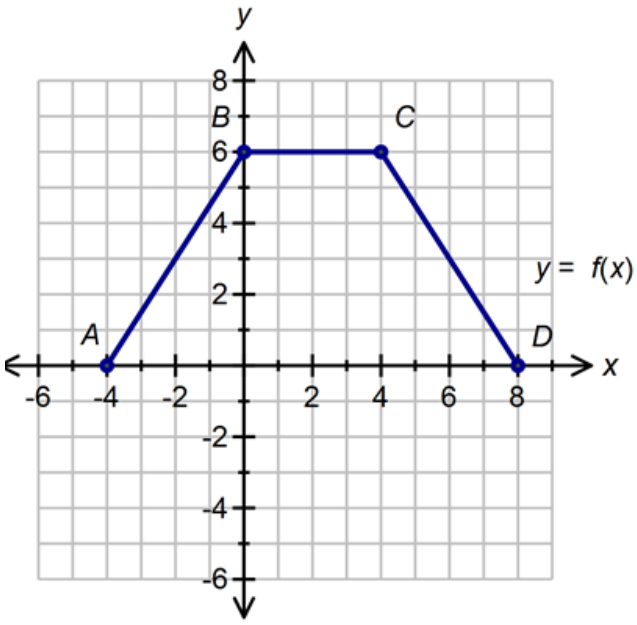
C(6,0) ⟶ C'

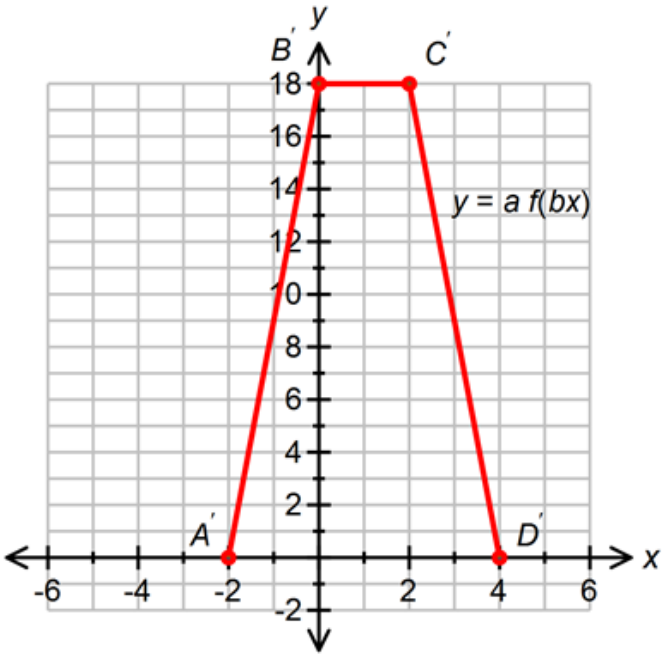
D(9,-1) ⟶ D'

E(12,0) ⟶ E'

Example 8

Determine the horizontal and vertical stretch and then write the equation of the transformed function in terms of $y = af(bx)$.





Domain

Range

Your Turn

Write the equation of the transformed function given the following mapping rule.

(i) $(x, y) \to (2x, 3y)$

(ii) $(x, y) \to \left(-\frac{1}{2}x, y\right)$

(iii) $(x, y) \to (-3x, -2y)$

Assign p.29-31 #5a-d, 6ab, 7ac, 9bef,14abc