CHAPTER 1

Function Transformations

Transformation

↳ the graph of a function may be changed either by shifting, stretching or compressing, or applying a reflection.

Types of transformations

1. Reflection on the x-axis → $y = -f(x)$ or $-y = f(x)$
2. Reflection on the y-axis → $y = f(-x)$
3. Vertical Stretch → $y = af(x)$ or $\frac{1}{a}y = f(x)$
4. Horizontal Stretch → $y = f(bx)$
5. Horizontal Translation → $y = f(x-h)$
6. Vertical Translation → $y = f(x)+k$ or $y-k = f(x)$

Combined Transformations

↳ Reflect, Stretch Translate

$$y = a(f(b(x-h))+k$$

or

$$\frac{1}{a}(y-k) = f(b(x-h))$$

Lesson 1.1: Vertical and Horizontal Translations

Translation:

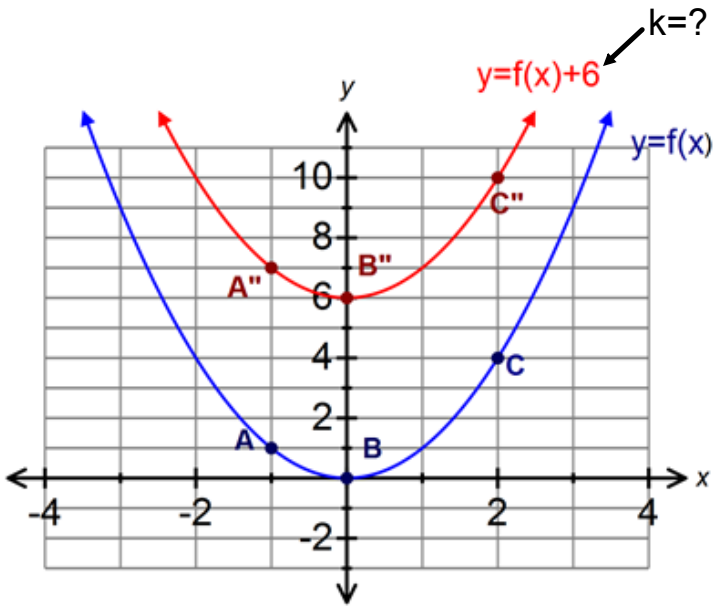
the graph of the function moves: up/down (vertical translation)
right/left (horizontal translation)

Vertical Translation (k): $y = f(x) + k$

affects y-value

two cases $k > 0$, $k < 0$

Case 1: $k > 0$



key points

A (-1, 1) → A"

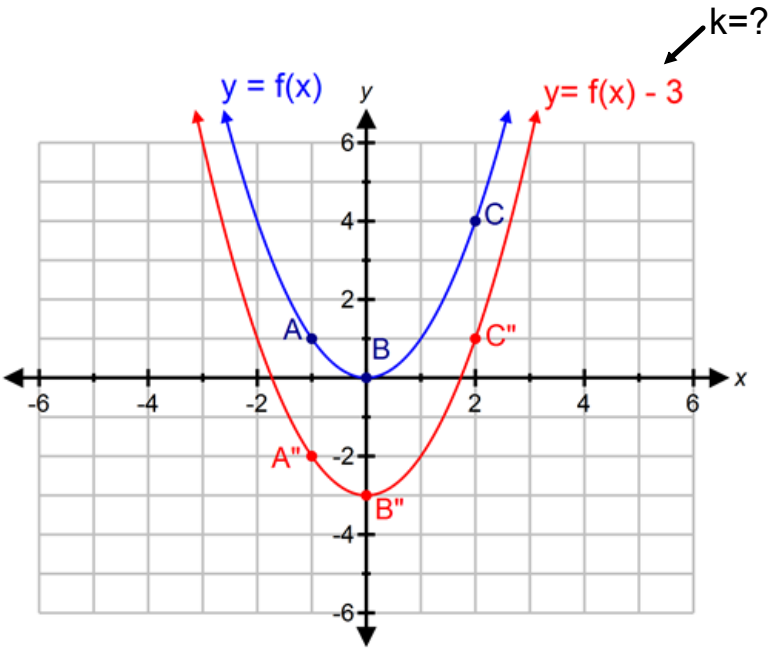
B (0, 0) → B"

C (2, 4) → C"

(i) How do the coordinates of the point change?

(ii) Write the mapping rule.

Case 2: $k < 0$



A (-1, 1) → A" (-1,-2)

B (0, 0) → B" (0,-3)

C (2, 4) → C" (2,1)

(i) How do the coordinates of the point change?

(ii) Write the mapping rule.

Summary:

$y = f(x) + k$ vertical translation

$k > 0$ the translation is up

$k < 0$ the translation is down

Mapping Notation: $(x, y) \rightarrow (x, y + k)$

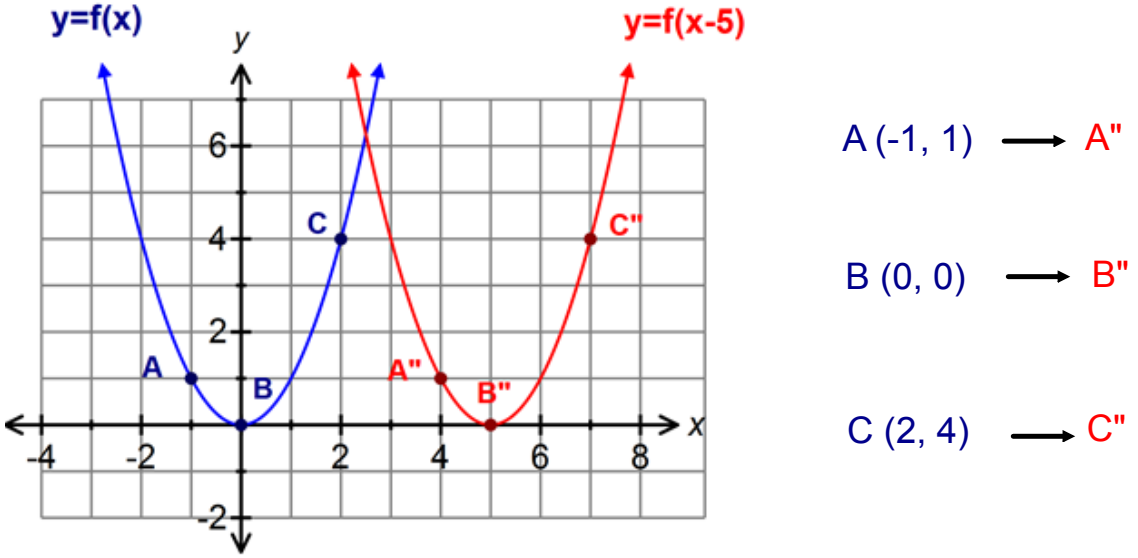
Horizontal Translation (h): $y = f(x - h)$

affects x-value

two cases $h > 0$, $h < 0$

Case 1: $h > 0$

$h =$



A (-1, 1) → A"

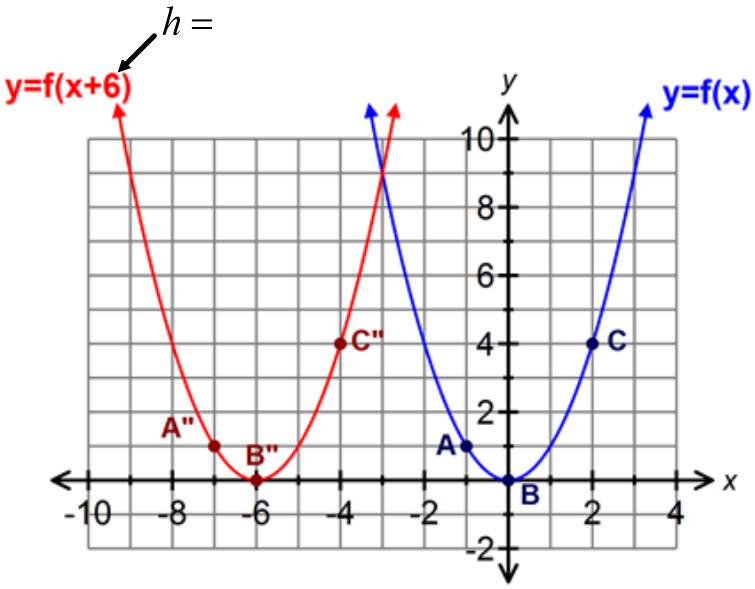
B (0, 0) → B"

C (2, 4) → C"

(i) How do the coordinates of the point change?

(ii) Write the mapping rule.

Case 2: $h < 0$



A (-1, 1) → A" (-7,1)

B (0, 0) → B" (-6,0)

C (2, 4) → C" (-4,4)

(i) How do the coordinates of the point change?

(ii) Write the mapping rule.

Summary:

$y = f(x - h)$ Function

$h > 0$ Translation is to the right

$h < 0$ Translation is to the left

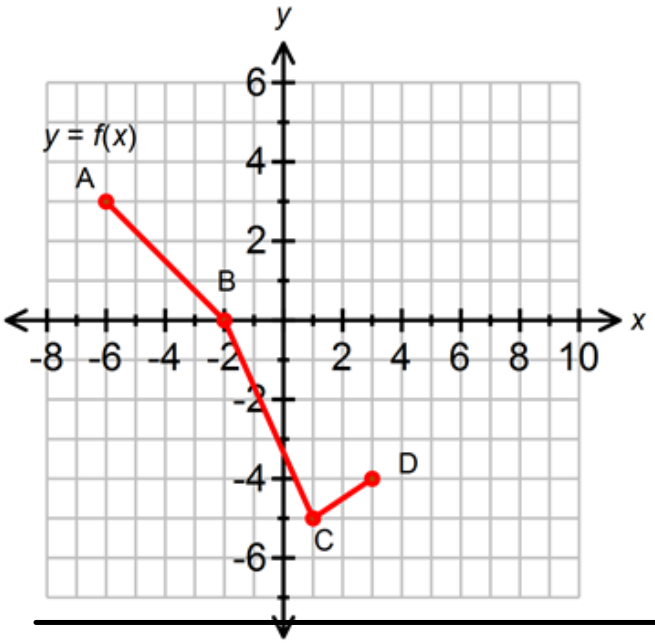
Mapping Notation: $(x, y) \to (x + h, y)$

Example 1

Given the graph of $y = f(x)$, identify the parameters h and k and create a mapping rule for each of the transformations below.
Graph the transformed function.

$(i)\ y = f(x-6)-2$

$h =$ $k =$



$(x, y) \rightarrow$

A (-6, 3) ⟶ A"

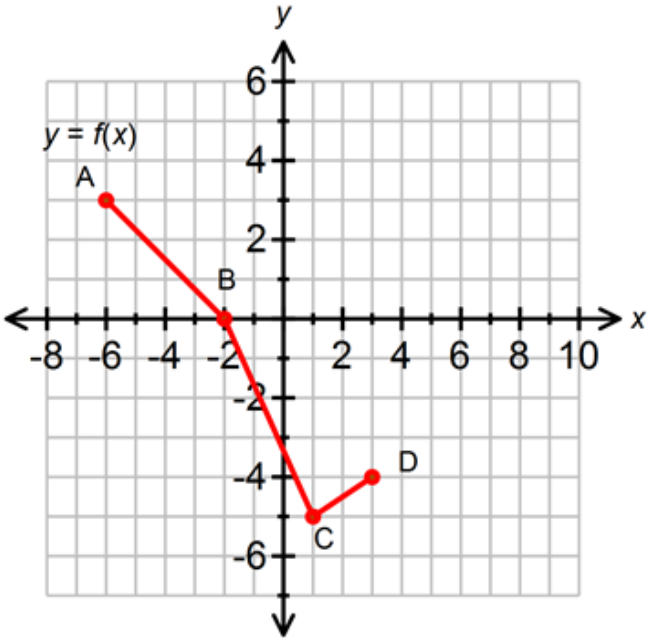
B (-2, 0) ⟶ B"

C (1, -5) ⟶ C"

D (3, -4) ⟶ D"

$(ii)\ y-3 = f(x+2)$

$h =$ $k =$



$(x, y) \rightarrow$

A (-6, 3) ⟶ A"

B (-2, 0) ⟶ B"

C (1, -5) ⟶ C"

D (3, -4) ⟶ D"

Example 2

For each transformation, identify the values of h and k and write the equation of the transformed function $y = f(x-h)+k$

$(i)\ f(x) = |x|$ translated 4 units to the left and 6 units up.

$(ii)\ f(x) = \frac{1}{x}$ translated 1 units to the right and 3 units down.

Example 3

What horizontal translation is applied to $y = x^3 - 2x^2 + x - 2$
if the translation image graph passes through the point (5,10) ?

Assign P.12-14 #1a-e, 3a-d, 4a, 5ac, 6-8, 11ab