

1. Which of the following represents the equation of $y = f(x)$ after it has been shifted 2 units to the right and 1 unit down?

(A) $y = f(x-2) - 1$

B) $y = f(x-2) + 1$

C) $y = f(x+2) + 1$

D) $y = f(x+2) - 1$

$HT = 2$
 $VT = -1$

2. The graph of $y = f(x)$ has been reflected across the y-axis and stretched vertically by a factor of $\frac{1}{2}$. Which of the equations would describe the new graph?

A) $y = -f(2x)$

B) $y = 2f(-x)$

C) $y = -\frac{1}{2}f(x)$

(D) $y = \frac{1}{2}f(-x)$

$y = f(-x)$

$VS = \frac{1}{2}$

3. The graph of $y = f(x)$ has a domain of $-2 \leq x \leq 6$ and a range of $0 \leq y \leq 10$. Which of the following would best describe the domain and range for $y = -f(2x) + 1$?

A) $D: -2 \leq x \leq 6$
 $R: -9 \leq y \leq 1$

B) $D: -6 \leq x \leq 2$
 $R: 1 \leq y \leq 11$

(C) $D: -1 \leq x \leq 3$
 $R: -9 \leq y \leq 1$

D) $D: -3 \leq x \leq 1$
 $R: 1 \leq y \leq 11$

$HS = \frac{1}{2} \leftarrow$ affects domain

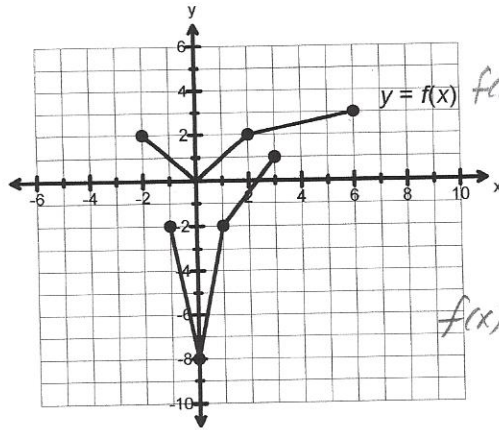
$VT = 1 \leftarrow$ affect range

reflection over x axis \leftarrow affects range

$D: -2 \leq x \leq 6 \Rightarrow -1 \leq x \leq 3$

$R: 0 \leq y \leq 10 \Rightarrow -10 \leq y \leq 0 \Rightarrow -9 \leq y \leq 1$

4. Which equation best describes the transformations that have been applied to $y = f(x)$ as shown in the graph?



A) $y = 3f\left(\frac{1}{2}x\right) - 8$

B) $y = 3f(2x) - 8$

C) $y = \frac{1}{3}f(2x) + 8$

D) $y = \frac{1}{3}f\left(\frac{1}{2}x\right) + 8$

$f(x) D: [-2, 6] = 8$
 $D: [-1, 3] = 4$

$HS = \frac{4}{8} = \frac{1}{2}$

$f(x) R: [0, 3] = 3$
 $R: [-8, 1] = 9$

$VS = \frac{9}{3} = 3$

$P(6, 3) \rightarrow (3, 9)$
HS, VS

$(3, 9) \rightarrow (3, 1) \Rightarrow VT = -8$

5. The graph of a function has a point with coordinates $(a, -b)$. If the graph has been reflected about the x-axis, horizontally stretched about the y-axis by a factor of 2 and translated 5 units down, what would the coordinates of the image point be?

A) $(a, 2b - 5)$

B) $(2a, b - 5)$

C) $(-2a, b - 5)$

D) $(-a, 2b - 5)$

$(a, -b) \rightarrow (a, b) \rightarrow (2a, b) \rightarrow (2a, b - 5)$
Reflection on x-axis

6. Given the point $(0, 5)$ is on a graph. For which of the following transformations would it remain an invariant point?

A) reflection in the x-axis, vertical stretch

B) reflection in the y-axis, horizontal stretch \Leftarrow affects x coordinate

C) horizontal translation

D) vertical translation

which is 0

7. What is the mapping rule for $y = -f\left(\frac{2}{3}x+4\right)-3$? $y = -f\left(\frac{2}{3}(x+6)\right)-3$

A) $(x, y) \rightarrow \left(\frac{2}{3}x-4, -y+3\right)$

B) $(x, y) \rightarrow \left(\frac{3}{2}x-4, -y+3\right)$

C) $(x, y) \rightarrow \left(\frac{2}{3}x+6, -y-3\right)$

D) $(x, y) \rightarrow \left(\frac{3}{2}x-6, -y-3\right)$

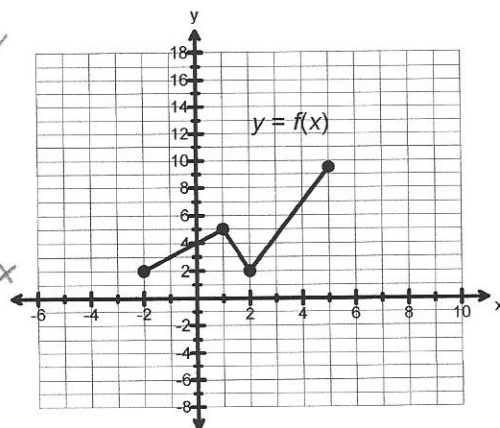
8. For the graph of $y = f(x)$ shown, which of the following statements is **not** true?

A) $(0, 4)$ will be an invariant point for $y = f(-x)$ *reflection on y*
 $(0, 4) \rightarrow (0, 4)$

B) $(2, 2)$ will be an invariant point for $y = f^{-1}(x)$ *inverse*
 $(2, 2) \rightarrow (2, 2)$

C) $(0, 4)$ will be an invariant point for $y = -f(x)$ *reflection on x*
 $(0, 4) \rightarrow (0, -4)$

D) There are no invariant points for $y = -f(x)$



9. What is the equation for the image graph of $y = f(x)$ as shown?

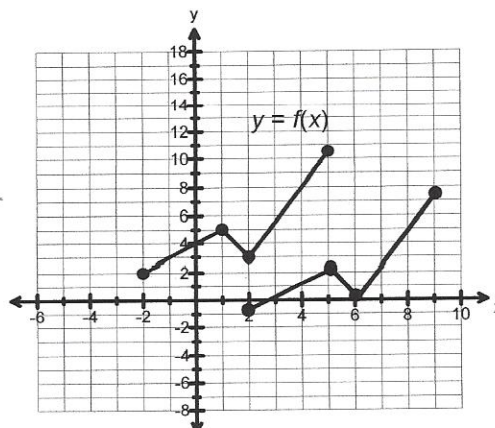
A) $y = f(x-4)+3$

B) $y = f(x+4)+3$

C) $y = f(x+4)-3$

D) $y = f(x-4)-3$

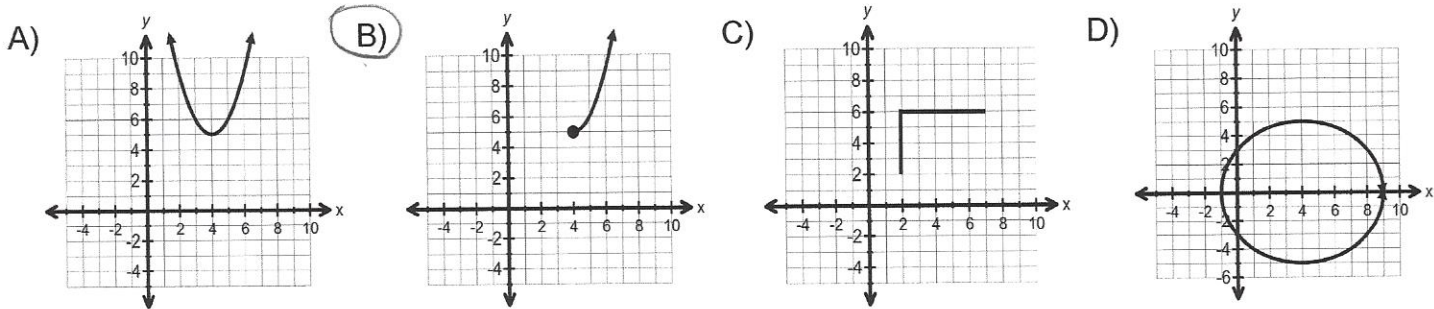
*4 Right
3 down*



10. If a function has a point with coordinates $(a, -b)$, what are the coordinates of a point on the graph of $y = f^{-1}(x)$? *switch*

- A) $(a, -b)$ B) $(b, -a)$ **C) $(-b, a)$** D) $(-a, b)$

11. Which of the graphs shown is the graph of a function which will have an inverse that is a function?



→ pass the horizontal line test

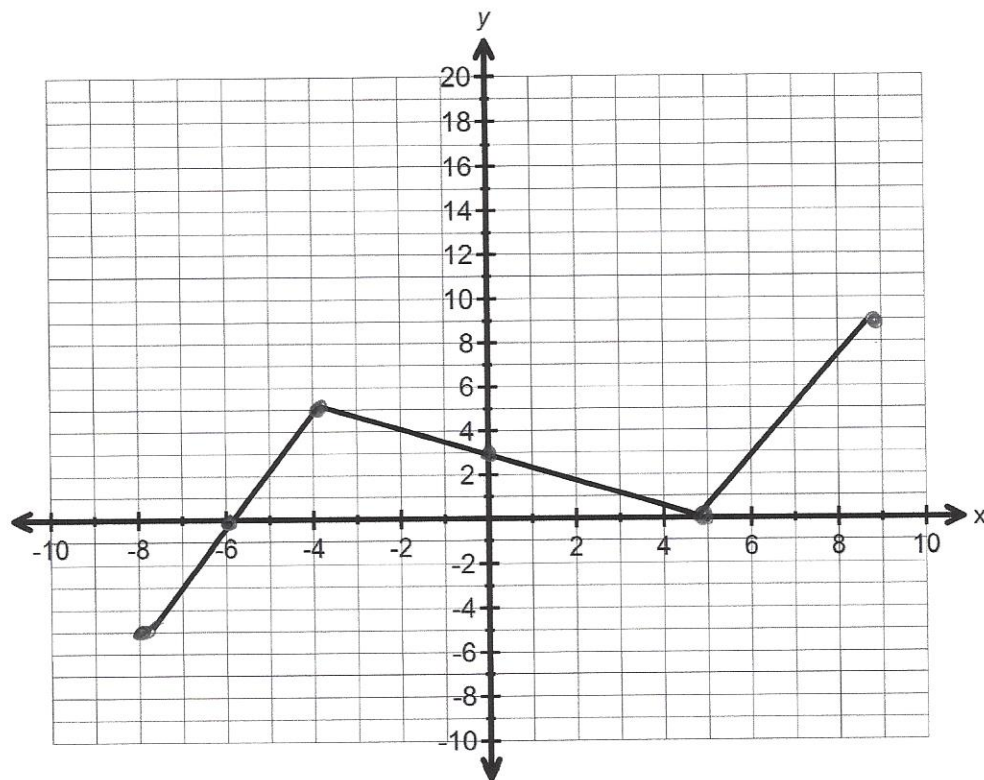
12. Given $y = f(x)$ has been transformed by a reflection in the x-axis, a horizontal stretch of 4, a horizontal translation of 1 unit left and a vertical translation of 2 units up, what is the new equation for the image graph?

- A) $y = -f(4x+1)+2$ B) $y = f(-4(x-1))+2$
C) $y = -f\left(\frac{1}{4}(x+1)\right)+2$ D) $y = f\left(-\frac{1}{4}x+1\right)+2$

$$\begin{aligned}
 y &= f(x) \\
 y &= -f(x) \\
 y &= -f\left(\frac{1}{4}x\right) \\
 y &= -f\left(\frac{1}{4}(x+1)\right)+2
 \end{aligned}$$

13. Given the graph $y = f(x)$ shown, sketch the graph for each of the following and state any invariant points

i) $y = 2f(-3(x+1)) - 2$



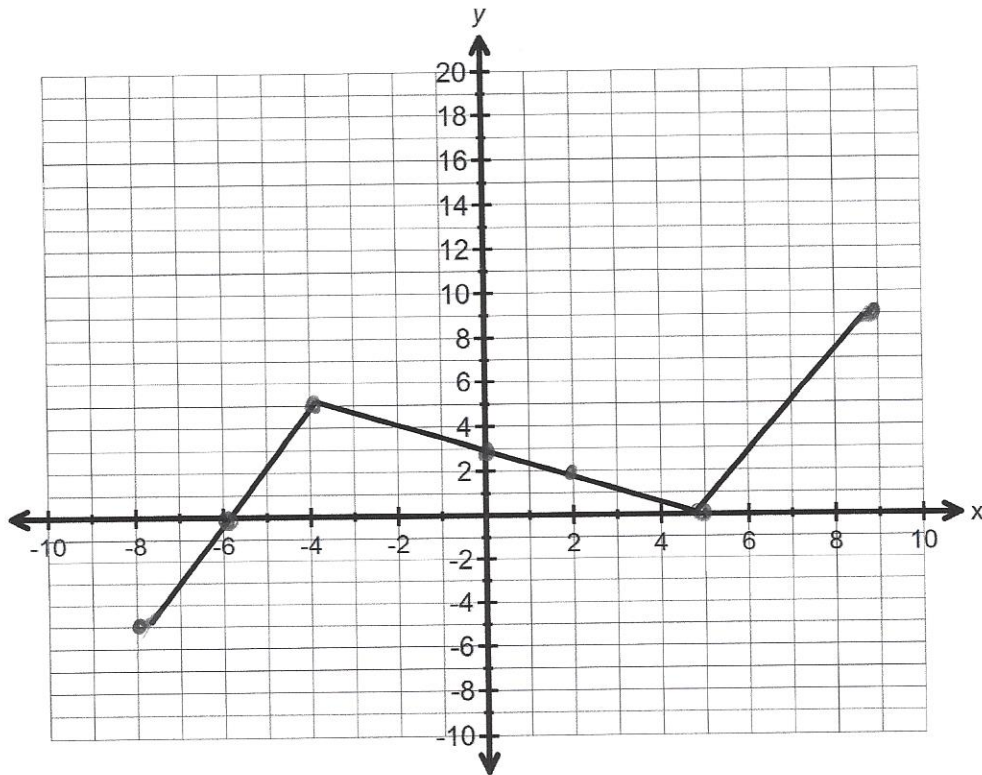
$$(x, y) \rightarrow \left(-\frac{1}{3}x - 1, 2y - 2\right)$$

x	y
-8	-5
-6	0
-4	5
0	3
5	0
9	9

→

x	y
$\frac{5}{3}$	-12
1	-2
$\frac{1}{3}$	8
-1	4
$-\frac{8}{3}$	-2
-4	16

ii) $y = f^{-1}(x)$



$(x, y) \rightarrow (y, x)$

x	y
-8	-5
-6	0
-4	5
0	3
5	0
9	9

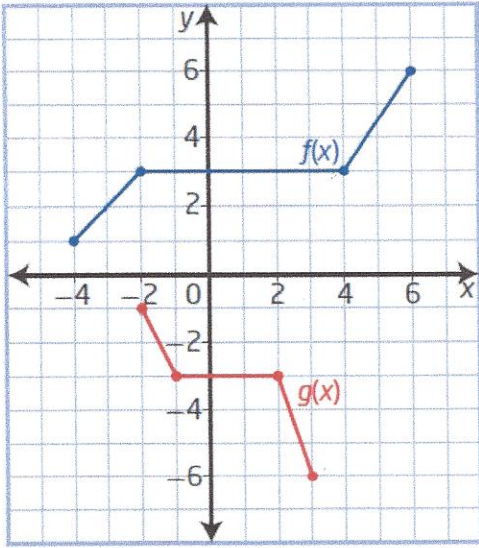
→

- $(-5, -8)$
- $(0, -6)$
- $(5, -4)$
- $(3, 0)$
- $(0, 5)$
- $(9, 9)$

invariant points $(2, 2)$ and $(9, 9)$

14. Determine the equation for the image of $y = f(x)$.

A)



reflection over x-axis

$f(x)$: Domain $[-4, 6] \Rightarrow 10$

$g(x)$: Domain $[-2, 3] \Rightarrow 5$

$HS = \frac{5}{10} = \frac{1}{2}$ $b = 2$

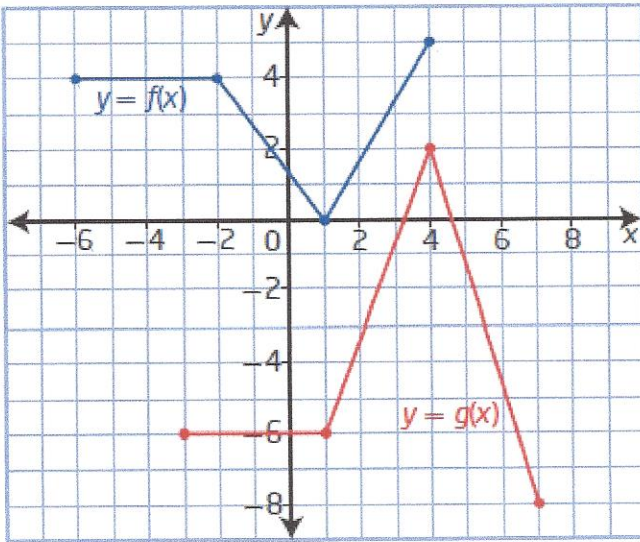
$f(x)$: Range $[1, 6]$

$g(x)$: Range $[-1, -6]$

Same $VS = 1$

$$y = -f(2x)$$

B)



$f(x)$: Domain $[-6, 4] = 10$

$g(x)$: Domain $[-3, 7] = 10$ $HS = 1$

$f(x)$: Range $[0, 5] = 5$

$g(x)$: Range $[-8, 2] = 10$

$VS = \frac{10}{5} = 2$ $a = 2$

reflection over x axis

$f(x)$ $(4, 5) \Rightarrow (4, -10)$

$(4, -10) \xrightarrow{VT = +2} (4, -8)$

$\xrightarrow{HT = +3} (7, -8)$

$$y = -2f(x-3) + 2$$

15. Algebraically determine the inverse of $f(x) = (x+1)^2 + 4$. $\checkmark (-1, 4)$ opens up
 State the restricted domain for the relation so that the inverse is a function.

$$y = (x+1)^2 + 4$$

$$x = (y+1)^2 + 4$$

$$(x-4) = (y+1)^2$$

$$\pm\sqrt{x-4} = y+1$$

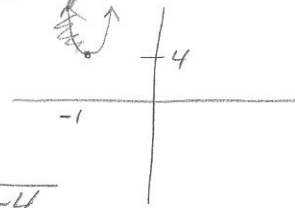
$$y = -1 \pm \sqrt{x-4}$$

Restriction

$$x \geq -1 \quad f^{-1}(x) = -1 + \sqrt{x-4}$$

or

$$x \leq -1 \quad f^{-1}(x) = -1 - \sqrt{x-4}$$



16. The graph of $y = f(x)$ is transformed to produce the graph of $y = -5f(2x-6) - 1$. What is the horizontal translation? If the point $(-4, 10)$ lies on the graph of $f(x)$ what is the image point on the graph of $y = -5f(2x-6) - 1$.

$$y = -5f(2(x-3)) - 1 \quad HT = 3 \text{ right}$$

$$(x, y) \rightarrow \left(\frac{1}{2}x + 3, -5y - 1\right)$$

$$(-4, 10) \rightarrow (1, -51)$$

17. Explain how the transformations described by $y = f\left(\frac{1}{2}x + 1\right)$ and $y = f\left(\frac{1}{2}(x+1)\right)$ are similar and how are they different.

$$y = f\left(\frac{1}{2}x + 1\right)$$

$$y = f\left(\frac{1}{2}(x+2)\right)$$

↓

$$HS = 2$$

$$HT = 2 \text{ left}$$

$$y = f\left(\frac{1}{2}(x+1)\right)$$

$$HS = 2$$

$$HT = 1 \text{ left}$$